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Quasidiffusion effects in fast gravitational flows of cohesionless particles of granular matter

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Abstract: The study analyzes the physical mechanisms behind the quasi-diffusive separation of cohesionless spherical particles in thin-layer fast gravity flows on a rough chute with substantial structural and kinematic parameter nonuniformity, with their complex size and density discrepancies. Studies have been conducted into alternative conditions of quasi-diffusive interaction of particles in a fast gravitational flow on a rough chute, which are defined by the dominance of the particles' relative velocities in the direction of gravitational shear or their chaotic fluctuations in the interaction. It has been found that the intensity of the quasi-diffusive separation flux is in direct dependence on the particle collision frequency, which, in the general case of gravity flows of granular matter, is determined at the dominant value of the component of the relative shear velocity of particles and depends to a lesser extent on the velocity of their chaotic fluctuations. In non-ordinary conditions of fast gravity flow, which are formed in the flow of nonsmooth elastic particles in its upper part, called the "cloud" of particles, the frequency of particle collisions is determined at the dominant value of the velocity of their fluctuations. It is found that in thin-layer fast gravity flows the effect of quasi-diffusive separation due to structural nonuniformity of the flow can dominate over the segregation effect resulting from local nonuniformity of the medium.

Keywords: granular matter; fast gravitational flow; structural heterogeneity; quasi-diffusion separation; size and density separation; segregation; void fraction gradient.

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Квазидиффузионные эффекты в быстрых гравитационных потоках несвязных частиц зернистых материалов

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Аннотация: Проанализированы физические механизмы квазидиффузионной сепарации когезионно несвязанных сферических частиц при комплексном их различии по размеру и плотности в тонкослойных быстрых гравитационных потоках на шероховатом скате с высокой неоднородностью структурно-кинематических параметров. Исследованы альтернативные условия квазидиффузионного взаимодействия частиц в быстром гравитационном потоке на шероховатом скате, отличающиеся доминированием при взаимодействии либо относительной скорости частиц в направлении гравитационного сдвига, либо скорости их хаотических флуктуаций. Установлено, что интенсивность потока квазидиффузионной сепарации находится в прямой зависимости от частоты столкновения частиц, которая в общем случае гравитационных течений зернистых материалов определяется при доминирующем значении компоненты относительной сдвиговой скорости частиц и в меньшей степени зависит от скорости их хаотических флуктуаций. В неординарных условиях быстрого гравитационного течения, которые формируются в потоке гладких упругих частиц в верхней его части, называемой «облаком» частиц, частота столкновений частиц определяется при доминирующем значении скорости их флуктуаций. Установлено, что в тонкослойных быстрых гравитационных потоках эффект квазидиффузионной сепарации, обусловленный структурной неоднородностью потока, может доминировать над эффектом сегрегации, являющимся следствием локальной неоднородности среды.

Ключевые слова: зернистый материал; быстрый гравитационный поток; структурная неоднородность; квазидиффузионная сепарация; сепарация по размеру и плотности; сегрегация; градиент доли пустот.

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1. Introduction

Granular matter, whose particles have no significant cohesive interactions, is frequently used in industrial technologies (chemistry, construction, food, mining, agriculture, etc.), in everyday life (cereals, legumes, nuts, salt, sugar, spices, etc.) and in the natural environment (sands, gravel, pebbles, rock fragments, etc.). Such materials look quite ordinary when they are considered at the level of individual particles, but at a closer look, they reveal original, mesoscopic properties, especially, when they are analyzed as a set of particles under certain interaction conditions [1, 2]. For example, depending on the interaction conditions, quartz sand particles can act as a solid base for building structures, as a liquid in an hourglass, and migrate as a gas of solid particles in the process of dune movement.

The most common form of particle interaction is their interaction under shear flow conditions of granular media. In this connection, for example, the piston flow of granular media is perceived as an idealized version of the flow.

In natural phenomena and technological processes in industry and agro-industrial complex, shear flows of granular matter most often occur in the form of fast gravitational flows. One of the main features of such flows is that particles interact intensively with each other and are involved in transverse mass transfer accompanied by quasi-diffusive effects of separation and mixing [3, 4]. The above effects are so significant that they can significantly influence the kinetics of the processing of granular matter and, moreover, can be used as a basis for the development of technological processes with original functional properties [5–7]. The analysis of literature sources shows that, despite the rather high scientific and applied significance of the study of quasi-diffusion effects under shear deformation of granular media, in most cases the studies are limited to modeling of quasi-diffusion mixing processes [8–10]. These circumstances explain that the physical mechanisms of quasi-diffusion effects of particle separation in fast gravitational flows of granular matter are the main object of the present study.

2. Analysis of methods for modeling separation effects in gravity flows of granular matter

The analysis of studies on modeling of separation dynamics in gravity flows of granular matter [5, 6, 8, 10] shows that in most of them the interaction of particles in the flow is modeled in the absence of attention to the structural nonuniformity of the flow. This is explained by the fact that in many cases of modeling dispersed media are analyzed in the state of gravitational shear at high and relatively stable values of pressure and low gradients of shear rate, when the assumption of structural uniformity of the medium can be quite correct. In addition, the analysis of gravitational flow mainly under conditions of its structural uniformity is explained by the great problems of identification of microstructural characteristics of fast gravitational flow of particles in states of unusual gas and liquid properties [2–4, 11].

The separation effects due to spatial nonuniformity of shear flows of granular matter were among the first to be identified by Stephens and Bridgewater [12]. Using an annular shear cell, the authors investigated the distributions of nonuniform particles in failure zones, where high shear rate gradients and void fractions result from intense particle interaction. The flow conditions formed in the failure zones of granular media significantly affect the kinetics of technological processes.

When studying the effects of separation and mixing of particles in the above-mentioned zones, the authors [12] found an unusual effect of movement of large particles of the mixture, which they called migration. The authors explained migration qualitatively as the effect of movement of large particles in the direction of the shear strain rate gradient. However, a sufficiently deep analysis of the physical mechanism of the detected effect was hindered by the unfavorable conditions for the analysis of obtaining measurement data. Firstly, problems in the analysis arose due to the highly complex shape of the shear flow, which resulted in the manifestation of difficult to estimate nonuniformity of the shear rate in the three-dimensional volume of the shear cell, for example, in the radial direction of the annular failure zone [12]. Secondly, and more importantly, a comprehensive analysis of the migration phenomenon was hindered by the limited amount of measurement information due to the lack of an opportunity to evaluate the influence of the structural nonuniformity of the failure zone on the effect in the form of the distribution of the void fraction or solid phase concentration in its volume. Therefore, the authors in [12] faced the problem of explaining the unusual distributions of mustard seed and polystyrene granules in the shear cell.

In addition, the simplest verification of the proposed migration mechanism by testing it on the results of studies of separation in gravity flow on a chute (e.g., Fig. 1*a*) does not make it possible to explain the opposite directions of migration of large particles near the bottom and open surface of the flow. When the shear rate varies monotonically along the bed thickness, large particles are displaced in the flow regardless of the shear strain rate gradient [13]. At the same time, it is important to note an obvious contradiction between the migration mechanism and the dispersion pressure mechanism proposed in [14], and according to which large particles move in the direction opposite to the shear strain rate gradient. Thus, with the unity of views on the driving force of separation due to the nonuniformity of the shear strain rate, the researchers in [12] and [14] determine the opposite direction of the process.

As a result, in the absence of a single abstract judgment regarding the physical nature of separation effects, the assumption of homogeneity of the characteristics of the gravitational flow structure is accompanied by serious problems in the mathematical description of the process. Consequently, it is often accompanied by the necessity to use different separation mechanisms in modeling the process depending on the distinctive properties of particles in invariant flow conditions. For example, when modeling the dynamics of separation of particles of different densities in a gravitational flow [15], the ascent mechanism is used, which formally reproduces the process of behavior of bodies with large and small densities in a liquid. At the same time, when describing the separation of particles by size under similar conditions, the mechanism of partial stress relaxation is used in [10].

In accordance with the mechanism of partial stresses, the stresses generated by gravitational shear are distributed on particles nonuniformity, if the latter are unequal in size. It is assumed that the stress distribution on nonuniform particles follows some law, according to which the stresses on particles of a certain size are proportional to some partial stress coefficient. It is important that in this case the values of the coefficients of the components of the particle mixture are not equal to their relative fraction [10]. In this case, the driving force of the process is expressed as a gradient of lithostatic pressure due to gravitational influence. It is assumed that particles of large size tend to generate partial stresses, whose share in the total stresses is greater than their concentration in the mixture, and are forced toward the lithostatic pressure gradient, i.e., toward the open surface of the flow [16]. Accordingly, small-sized particles generate partial stresses in the flow, the fraction of which in the total stresses is less than their volume fraction in the mixture. Such particles move in the direction of the lithostatic pressure gradient, i.e., to the lower layers of the flow, where the stresses have the maximum value.

The authors of relatively recent studies [8, 16], analyzing the physical mechanisms of buoyancy [15] and partial stress relaxation [10, 17], note their rather high flexibility, and conclude that they have a serious drawback, which is a consequence of the lack of attention to the separation effects initiated by the nonuniformity of shear flow parameters. Among such parameters, the authors in [8] emphasize the shear rate and granular temperature, which, in general, vary in the shear flow volume over a wide range. The necessity to consider the effects of spatial heterogeneity of shear flow characteristics on the separation dynamics was justified by many researchers of the separation process in gravity flows of granular media [8, 13, 16].

Assuming that the size and density separation proceeds according to different physical mechanisms, the authors in [8] considered it rational to divide the local stresses into two components: one component is contact stresses, and the other one is kinetic stresses. In this case, they assume that the size separation occurs under the action of the driving force caused by the action of kinetic stresses and expressed in the form of their gradient under the condition of disproportionate distribution of stresses on particles of different sizes. Kinetic stresses are stresses that are

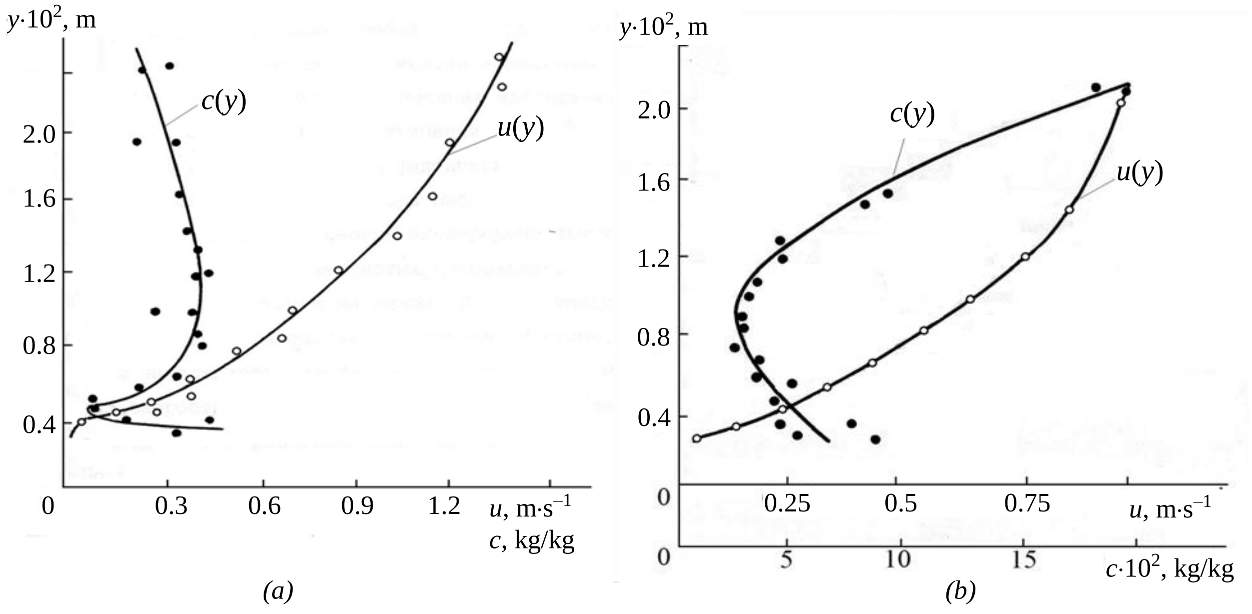

Fig. 1. Profiles of velocity $u(y)$ and concentration $c(y)$ of particles in flows on a rough chute in binary mixtures:
a) large particles in a mixture of steel balls;
b) silica gel granules in a mixture with equally sized granules of superphosphate

generated in granular matter under conditions of rapid shear deformation as a result of transfer of impact impulses by colliding particles across the shear surface.

However, if we take into account that the intensity of impulse interaction between particles is proportional to the shear rate, we should conclude that the directions of gradients of kinetic stresses and shear rate coincide. Thus, a certain physical analogy between the mechanisms of migration [12] and partial kinetic stresses [8, 16] is revealed, and the critical remarks made earlier with respect to the migration mechanism largely apply to the mechanism of partial kinetic stresses as well.

According to [8], density separation proceeds under the action of contact stresses in accordance with the previously presented buoyancy mechanism [15]. The driving force of separation in this case is gravitational influence under conditions of disproportionate distribution of contact stresses on nonuniform particles. It is important to pay attention to the fact that when developing the separation mechanisms due to spatial inhomogeneity of shear flow, the authors limit themselves to the case of particle size difference, considering that spatial inhomogeneity of shear flow does not affect the separation by density. The validity of this approach is refuted by experimental data [13] (e.g., Fig. 1*b*), which confirms the effect of spatial inhomogeneity of flow on density separation.

Thus, the process of separation by size and density is described [8] through separate physical mechanisms [10] and [15], each of which is adapted for an autonomous description of the separation effect with respect to a certain distinctive property of particles.

Based on the corresponding physical representations, a mathematical model of particle separation by size and density was developed in [8]. The mathematical model is based on the equation describing the dynamics of distribution of nonuniform particles in a flow with nonuniform shear strain rate

$$\rho^i\left(v^{c,i}-v\right)=\frac{(\psi^{c,i}-\psi^{k,i})}{c_D}\frac{\partial\sigma_{yy}^k}{\partial y}+$$
$$+\frac{(\phi_m^i-\psi^{c,i})}{c_D}\rho g\cos\zeta-D\frac{\partial\rho^i}{\partial y}, \qquad (1)$$

where ρ, ρ^i are bulk density of granular medium and particles of the i-th component, respectively; g is gravitational acceleration; v, v^i are velocity of granular medium and particles of the i-th component in the direction of y separation, respectively; ϕ_m^i is concentration of the i-th component; $\psi^{c,i}, \psi^{k,i}$ are coefficients of partial contact and kinetic stresses for the i-th component of the mixture, respectively; σ_{yy}^k is kinetic stress; ζ is flow slope angle to the horizon; D is mixing coefficient; c_D is drag coefficient in the process of particle separation.

The partial stresses $\psi^{c,i}$, $\psi^{k,i}$ and the ϕ_m^i component concentration are expressed as relative dimensionless quantities.

According to the mathematical description (1), the concentration field of control particles of granular medium in shear flow is determined by the conjugation of the effects of particle transport by convection, separation and quasi-diffusive mixing, and the manifestation of separation effects is caused by the disproportionate distribution of two types of stresses on nonuniform particles. It is important to note that due to this approach, the description of the dynamics of the separation process is provided taking into account both the local nonuniformity of particles of the granular medium and the spatial inhomogeneity of kinematic characteristics in the shear flow of the matter.

However, verification of the mechanisms and kinetic dependences of the separation effects proposed in [8] due to both local and spatial inhomogeneity of the granular medium flow reveals their apparent contradictory nature. Verification was carried out by testing the mechanisms and kinetics based on the results of experimental studies presented in [13] (Fig. 1). Figure 1 shows experimental profiles of velocity and distributions of control particles in fast gravity flows of mixtures of particles of different sizes (Fig. 1*a*) and densities (Fig. 1*b*) on a rough chute. First of all, when. testing the model representations on which the separation dynamics equation (1) is based, it is important to pay attention to the experimental velocity profiles, which in both cases (Figs. 1*a* and 1*b*) have a parabolic shape with a monotonic change in the shear rate and its gradient directed toward the bottom of the flow. However, when the direction of the shear rate gradient in the flow is constant, different directions of the separation fluxes by size and density are observed. In the processes of separation by both size and density, the direction of movement of the particles of the control component at the bottom and at the open surface of the flows is opposite, which is impossible to explain from the position of model ideas about the mechanisms of separation by size and density, on which the mathematical description of the separation dynamics is based (1).

One of the reasons for such contradictions may be the lack of due attention to the structural nonuniformity of the granular medium flow when developing [8] the physical mechanism and mathematical model of separation. Given the fundamentally important role of shear rate nonuniformity with respect to the size separation kinetics, the assumption made does not seem to be correct enough. This conclusion is confirmed by many examples that show the presence of a pronounced dependence of the void fraction in the shear flow of a granular medium on the shear rate. Such examples are the results of experimental studies of the relationship between kinematic and structural parameters under conditions of both fast gravity flows on a rough chute and pseudoplastic shear deformations of granular matter [7, 13]. Since the volume fraction of particles in shear flow significantly affects [2] the conditions of their interaction and flow dynamics, the assumption of homogeneity of solid phase distribution in a flow with pronounced nonuniformity of shear rate, is one of the reasons for the inadequacy of modeling found in [8]. It is difficult to assume that the ratio of partial contact and kinetic stresses for particles of different size and density under conditions of structural inhomogeneity of the flow remains constant.

The lack of a sufficiently developed theoretical base does not allow for identification of a complex of kinetic parameters characterizing the intensity of separation and mixing effects and becomes the reason for the use of fitting coefficients. For example, according to the process dynamics equation (1), the number of such coefficients can be equal to four, if we take into account that the kinetic characteristics of the equation are complex functions not only of the shear rate, but also of the void fraction, which is in a complex dependence on the shear rate.

The above situation indicates the relevance of analytical and experimental studies of the mechanisms and kinetic regularities of size and density separation effects in fast shear flows of particles, the failure zones of which are characterized by high gradients of kinematic and structural characteristics. The purpose of the present study is to analyze the results of a comprehensive study of size and density separation in particle flows on a gravity chute under conditions of a vivid manifestation of inhomogeneity of flow parameters in its failure zones. Due to the dislocation of failure zones at the base and surface of such flows, thin-layer gravity flows are the most suitable for studies. With a relatively large volume fraction of failure zones in thin-bed flows, conditions for intensive separation of particles under the effects of inhomogeneity of flow parameters are formed in most of them. It is interesting to note that, in many formal features, the flow conditions occurring in a thin-bed fast gravity flow are similar to the conditions of shear flow in a annular shear cell [12] and differ from the latter by more favorable conditions for analysis in a two-dimensional flow.

3. Research methods and model materials

The regularities of particle size and density separation under the effects of gravity flow inhomogeneity are investigated using an experimental-analytical method [7]. The method provides access to a set of the most important information on flow parameters and particle separation effects on a rough chute in the form of velocity profiles, void fraction and distributions of nonuniform particles.

Under this method, experimental information is obtained using a setup that is an open inclined channel of rectangular cross-section with smooth side walls and a rough bottom. The channel is set at an angle to the horizon with some excess of the angle of repose of the material. To ensure the conditions of sticking at the bottom of the flow and intensive shear deformations in its volume, the roughness value of the channel base is equal to half of the diameter of the largest particles of granular medium. To feed the material into the channel the unit is equipped with a metering feeder. A horizontal cuvette with transverse partitions dividing the cuvette into cells is installed under the threshold of channel on the plumb line. Cells are used to receive falling particles in the stationary mode of steady-state material flow to obtain the corresponding distribution functions of material and test particles. The stationary regime of steady-state flow is confirmed by the fact that the distributions remain unchanged when the length of the rough chute is increased in the experiments.

The set of experimental information necessary for the implementation of the method, in addition to the mentioned distribution functions, includes the time of filling the cells of the cuvette, the width of the channel and its slope angle, the geometric parameters of the bed on the chute and the height of placement of the discharge threshold above the cuvette. Based on the obtained information, the profiles $u(y)$, $\varepsilon(y)$ and $c(y)$ are determined by analytical solution of the system of equations including the material balance equation, the law of a free-falling body and the equation of state of the granular medium under conditions of rapid gravitational shear [7]. The solution is obtained assuming the condition of zero longitudinal velocity of particles at their contact with bottom roughness ($u = 0$ at $y = 0$).

In the analysis, in addition to the original data, the results of the study obtained earlier [13] using mixtures (Table 1) with complex particle size and density differences: 1) in size (mixture 1); 2) mainly in density (mixture 2); 3) in size and density (mixture 3) were used.

The dynamics of the concentration field of test particles in shear gravity flow of granular matter is defined [7, 13] as the resultant effect of particle transport by convection j_c, separation j_s, caused by factors of both local and spatial heterogeneity of the flow, and quasi-diffusive mixing of j_d component:

$$\frac{\partial(c\rho)}{\partial t}+\mathrm{div}\left(\vec{j}_c\right)=-\mathrm{div}\left(\vec{j}_s\right)-\mathrm{div}\left(\vec{j}_d\right), \tag{2}$$

where t is time; ρ is local value of bulk density, which is defined as a function of control particle concentration $c(x, y, t)$ under conditions of stationary distribution of void fraction in the flow $\varepsilon(y)$.

The kinetics of size and density segregation initiated by local flow inhomogeneity is modeled on the basis of the mechanism of shear flow segregation [7, 18, 19]. According to this mechanism, segregation is a consequence of the process of stress relaxation on particles – stress concentrators, which differ in properties from uniform particles of a conventionally homogeneous flow. In the relaxation process, an aggregate of particles is formed near the stress concentrator, and when interacting with it, the concentrator acquires a transverse momentum that initiates its displacement accompanied by a weakening of the stress state.

Table 1. Characterization of mixture matter

Matter	Mixture 1		Mixture 2		Mixture 3	
	Steel balls		Superphosphate	Silica gel	Superphosphate	Silica gel
Particle diameter, 10^3 m	6.6	7.0	+3.75 – 4.0		+3.75 – 4.0	+4.0 – 4.25
Weight concentration, %	56.6	43.4	95.6	4.4	95.6	4.4
Density, kg·m^{-3}	7850		1800	785	1800	785
Slope angle, deg	31		35		35	

The control particle contacts the aggregate in points, the coordinates of which are determined by methods of statistical mechanics with regard to the local value of solid phase concentration and the ratio of sizes of the test particle and the particles of a conventionally uniform medium forming the aggregate. Through the contact points passes the axis of rotation of the particle under the action of ΔM excessive momentum of gravity, friction and impact momentums in the direction of segregation. As a result, when the process driving force ΔM and the process kinetic coefficient K_S are used, it is possible to reflect the influence of the complex of flow parameters and the full complex of particle properties on the segregation kinetics, which is expressed in the following form

$$\vec{j}_s = K_S c \rho \Delta M. \tag{3}$$

Studies of the mechanism and kinetics of segregation [7, 18, 19] have contributed to the deepening of knowledge about its mechanism and kinetics and enabled to propose methods for determining the driving force and the segregation coefficient K_S. The results of the study indicate that for a wide range of particle properties of a certain material and its gravitational flow parameters, the coefficient K_S exhibits the properties of a kinetic constant.

It is worth noting that the action of the shear flow segregation mechanism is limited by the conditions of low values of the void fraction ($\varepsilon < 0.75$), allowing the formation of particle aggregate near the particle – stress concentrator [7, 18]. In this connection, it can be concluded that in the gravity flow regions with high values of the void fraction ($\varepsilon \geq 0.75$) the particles are in a state of intensive chaotic (quasi-diffusive) movements and, as a consequence, it seems possible to assume that the effects of their segregation are determined mainly by diffusion kinetics.

4. Results and Discussion

4.1. Mechanisms and kinetic regularities of quasi-diffusive separation by size and density due to structural nonuniformity of gravity flow of granular matter

Mathematical modeling of the process of separation by size and density in thin-bed flow of cohesionless particles on a gravity chute with pronounced inhomogeneity of flow parameters was carried out using equation (2). The equation was used to predict the dynamics of the concentration field of test particles under the action of a set of effects (separation, quasi-diffusion, convection). Taking into account the revealed shortcomings of the analyzed methods of modeling separation, its flow is represented by two components with fundamentally different nature of the driving force. To evaluate the degree of influence of the effects of nonuniformity of structural and kinematic parameters on the predictive properties of equation (2), we analyzed the possibilities of predicting experimental concentration profiles of control particles in mixtures 1 and 2 (Table 1) taking into account only the component of the separation flow formed under the action of the factor of local nonuniformity of the flow. The intensity of the named component of the separation flux is expressed (3) on the basis of the mechanism of shear flow segregation, which is described in the previous section of the paper. In accordance with the mechanism, the segregation flow is the result of formation and relaxation of local stress concentration centers.

The mathematical description of separation using the kinetic dependence (3) provides prediction of the dynamics of the concentration field of particles differing not only in size and density, but also in the full complex of properties during gravity flow. However, it is established that in some cases, mainly in conditions of thin-bed gravity flow, with their characteristic high inhomogeneity of flow parameters, there are problems of mathematical description of separation dynamics. Moreover, the problems of mathematical description are found with respect to separation by size and density.

The inadequacy of modeling is clearly manifested in the fact that the shear segregation mechanism cannot explain the separation flows of less dense particles in mixture 2 to the base and fine particles in mixture 1 to the flow surface. If we assume that the observed counter directions of separation of particles by size and density take place at unchanged direction of the shear rate gradient in the flow, then obvious contradictions of experimental data with the mechanisms of buoyancy and partial stresses are revealed [8, 12, 14, 15, 17]. The observed paradoxical counterflow separations in size and density initiated a comprehensive study of the separation effects due to the inhomogeneity of gravity flow parameters of granular media.

The separation effects due to the inhomogeneity of thin-bed gravity flows have been investigated by the experimental-analytical method [7, 13, 18, 19], the description of which is given in the previous section of the paper.

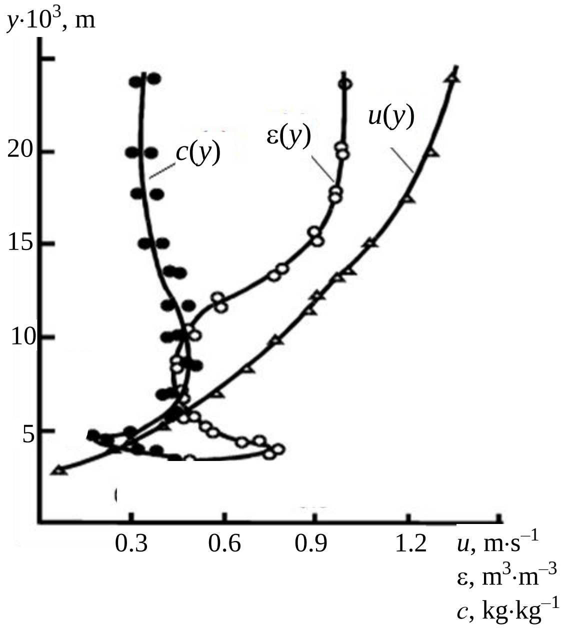

Fig. 2. Profiles of velocity $u(y)$, void fraction $\varepsilon(y)$ and large particle concentration distribution $c(y)$ in the fast gravity flow of mixture 1 (Table 1) on a rough chute

The structural and kinematic characteristics of the gravity flow of mixture 1 (Table 1) in the form of void fraction and velocity profiles are shown in Fig. 2. The same figure shows the distribution of large particles, which contain sufficient information about the separation effects that occur in the void zones in the flow on the rough slope [13]. The experimental data indicate an extremely high concentration of large-sized particles in the flow core, far exceeding their average concentration. Accordingly, in the directions from the flow core to its bottom and open surface, the concentration of small-sized particles increases significantly above the average value.

Experimental results and their discussion

The joint analyses of the profiles of void fraction and concentration distribution of large particles indicate that their shape can be considered as a mirror reflection of each other's characteristic features. It is obvious that for the profile of small particles distribution, a high analogy with the profile of void fraction can be observed.

Similar profiles $u(y)$, $\varepsilon(y)$ and $c(y)$ characterizing the gravity flow parameters and separation effects in the rarefied zones on the rough chute for mixture 2 (Table 1) are shown in Fig. 3. The results indicate that in the case of particle density differences, as well as in the case of particle size nonuniformity, the shape of the void fraction profile $\varepsilon(y)$ characteristic of steady-state fast gravity flow is observed. In the core of the flow a zone with the highest fraction of solid phase is formed, the content of which in the peripheral regions of the flow decreases with the formation of rarefied zones, especially near the open surface of the layer.

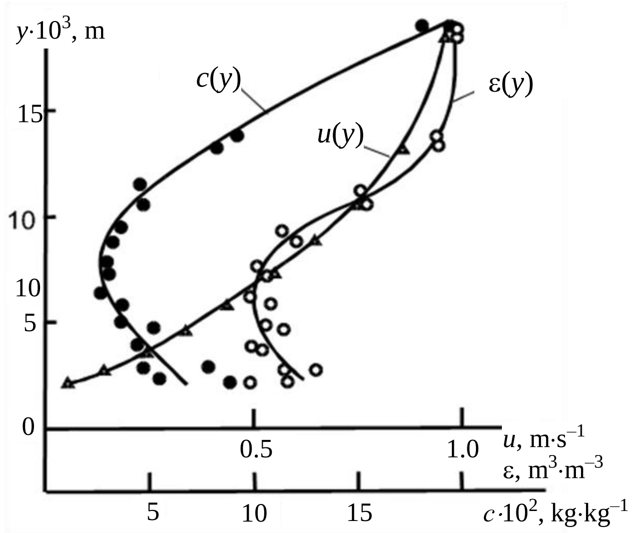

Fig. 3. Profiles of velocity $u(y)$, void fraction $\varepsilon(y)$ and low-density particle concentration distribution $c(y)$ in fast gravity flow of mixture 2 (Table 1) on a rough chute

The profile of particle distribution with low density $c(y)$ by characteristic features of the shape agrees with the profile of the void fraction $\varepsilon(y)$. The comparison of the profiles indicates a complete coincidence of the coordinates of the extreme values of the distribution functions of the void fraction $\varepsilon(y)$ and the concentration of test light particles $c(y)$. The minimum of the mentioned distribution functions is in the region of the core of the flow of granular matter on the rough chute. In the direction towards the bottom and the upper open surface of the flow, the values of these parameters increase significantly.

As a result of a complex analysis of the sets of profiles shown in Figs. 2 and 3 for flows of mixtures of particles nonuniform in size and density, it is possible to draw conclusions regarding the kinetic regularities of the observed separation effects.

Despite the unchanged direction of the shear rate gradient in the flow volume (in the direction of the bottom), in its regions near the open surface and the bottom, the size and density separation flows have opposite directions. It is noteworthy that particles with high inertial properties (larger and denser) in the process of separation move from the peripheral parts of the flow and concentrate in its central part.

Further abstraction of the experimental profiles shown in Figs. 2 and 3 suggests that, because of size and density separation, particles with large mass are concentrated in the core region of the flow with characteristically high values of void fraction. Correspondingly, there is a concentration of particles with low mass (small and low density) in the peripheral parts of the flow with characteristically high values of void fraction. In this connection it should be concluded that in the fast gravitational flow of granular medium there is an effect of structural nonuniformity, under the influence of which the separation flows of particles with small mass in the direction of the gradient of the void fraction and particles with large mass in the direction of the gradient of the volume fraction of solid phase are formed. In addition, when analyzing the variable values of the degree of curvature of profiles, it seems fundamentally important to pay attention to the importance of the void fraction gradients as a kinetic parameter that determines the intensity of the separation effect due to the inhomogeneity of shear flow parameters. This can be confirmed, for example, by density separation effects (Fig. 3), the high intensity of which can be explained by high values of the void fraction gradient $\varepsilon(y)$.

Among many extraordinary experimental results, the effect of moving of low-density particles from the core to the bottom of the flow attracts special attention (Fig. 3). It is important that in the steady-state flow regime the fraction of particles with low density increases towards the bottom until the concentration exceeds its average value in the mixture. The observed effect cannot be explained by using a set of previously analyzed separation mechanisms [8, 10, 12, 14, 15]. In this connection, there are sufficient grounds to develop the mechanism of separation as an effect of structural nonuniformity of rapid gravity flow of granular medium by analyzing the given experimental data.

A comprehensive analysis of a set of profiles $u(y)$, $\varepsilon(y)$, $c(y)$ (Figs. 2 and 3), characterizing structural-kinematic parameters and separation effects in fast gravitational flows of particles, gives grounds to assert that the movement of particles in the process of separation by size and density occurs under the action of the transfer potential, which has a direct relation to the gradient of the void fraction. The main type of interaction of particles in the rarefied zones of fast gravitational flows is their collision accompanied by the transfer of impact impulses. At chaotic exchange of impact impulses, particles differing in size and density will be in a state of chaotic fluctuations proceeding with different velocity of displacements. At collision of particles having different densities, the velocities of chaotic displacements for particles with low density, in general case, will be higher than for dense particles.

Collisions of particles at gravitational shear take place in confined conditions. In this connection, the total momentum acquired by a particle is determined in direct dependence on its surface area [13]. In this case, the average value of the fluctuation velocity of the *i*-th particle having diameter d_i and mass m_i is defined as: $V_i' \approx k d_i^2 m_i^{-1}$, where k is proportionality coefficient.

The nature of interaction of particles under conditions of fast gravitational shear has an obvious quasi-diffusive nature. At the difference in size and density, the fluctuation velocities and, accordingly, quasi-diffusion of particles will have different average values depending on their properties. In such a case, under conditions of structural nonuniformity of the shear flow, the quasi-diffusion separation effect is manifested, which has a physical nature with a high formal analogy with the nature of the molecular thermo-barodiffusion effect [20]. In the process of molecular thermo-barodiffusion, a diffusive separation flow of molecules with high fluctuation velocity is formed in the medium region, providing the possibility of fluctuations of molecules at large free path length. Molecules having a small velocity of chaotic movements are moved by the flow of separation in the medium region, in which the conditions for fluctuations of molecules at a small free path length are formed.

Under conditions of rapid shear deformation of granular medium in steady flow on a rough gravity chute, a zone with maximum values of solid phase fraction is formed in the central part of the bed. The size of the zone and the values of the solid phase fraction in it depend on the thickness of the bed and the value of the chute angle to the horizon. In this flow zone with the lowest values of the void fraction, conditions are created for the movement of particles inclined to quasi-diffusion with low velocity and amplitude of fluctuations. Consequently, according to the dependence of the fluctuation rate on the diameter and mass of particles ($V_i' \approx k d_i^2 m_i^{-1}$), conditions for quasi-diffusive fluctuations of particles with large mass are created in the central zone of the flow. At the same time, particles with small masses, which acquire large fluctuation velocities during collisions, participate in quasi-diffusive movements with large amplitude of oscillations and are displaced to the peripheral region of the flow, where conditions for fluctuations of particles with large free path length are formed.

4.2. Modeling of kinetics of quasi-diffusion separation process

This section of the paper presents variants of a mathematical model of quasi-diffusive separation by size and density for two idealized conditions of interaction of cohesionless spherical particles under rapid shear deformation of a granular medium with high structural nonuniformity. The analyzed conditions of particle interaction are of decisive importance in estimating the frequency of their collisions. In contrast to the previously presented studies [7, 13, 18], the present paper presents a comparative analysis of the variants of modeling the contact interaction of particles with the involvement of experimentally obtained information.

Variants of contact quasi-diffusion interaction of particles under conditions of rapid shear deformation are analyzed using the example of [13] of steady-state shear flow of a mixture of two components (components 1 and 2) having particles of diameter d_1 and d_2, density ρ_1, ρ_2, and mass m_1, m_2, with their concentration in the mixture c and $1 - c$, respectively. The mixture of particles is subjected to rapid shear deformation in the shear direction x (Fig. 4). It is assumed that under the action of a set of variable factors (deformation rate du/dy and lithostatic pressure), structural nonuniformity of the flow occurs, expressed in an increase in the fraction of voids in the y direction.

It is assumed that under conditions of steady-state shear flow of particles, the convection mechanism of transfer dominates in the process of their mixing in the direction of the shear. In the y direction, perpendicular to the direction of the shear, nonuniform particles change their position in the flow due to the manifestation of quasi-diffusion effects of particle interaction, accompanied by their mixing and separation.

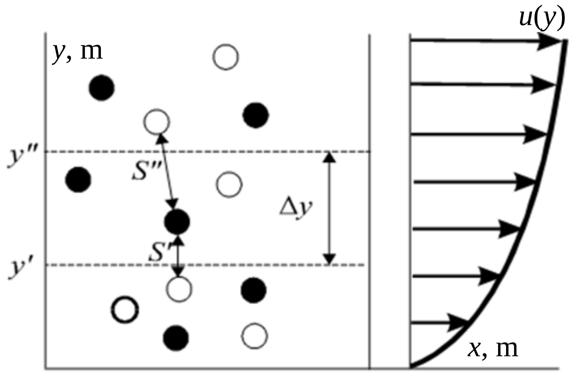

Fig. 4. To the mechanism illustration and formulation of kinetic regularities of quasi-diffusive separation by size and density due to structural nonuniformity of shear flow of particles

The latter assumption allows us to analyze the quasi-diffusive interaction of nonuniform particles in a fast shear flow under conditions of inhomogeneity of its structural and kinematic parameters, neglecting the effect of separation (segregation) due to the relaxation of stress concentrators. It should be noted that such conditions occur at high values of the void fraction ($\varepsilon \geq 0.75$), and in accordance with most physical mechanisms, segregation is a consequence of relaxation of contact stresses during the interaction of nonuniform particles in the flow.

In the process of analysis, an elementary layer bounded by the coordinates y' and y''. The surfaces bounding the layer are perpendicular to the y axis, and the layer thickness Δy satisfies the condition $\Delta y = y'' - y' << 2s$, where s is the distance between particles averaged over the volume of the layer [13].

The relative rate of quasi-diffusion of nonuniform particles q_{rel} necessary for the expression of the separation flux is determined under the condition $c(y) = \text{const}$ within the selected layer and the gradient of the quasi-diffusion transfer potential equal to 1. In accordance with the kinetic theory [20], the indicated flux is expressed in the following form:

$$q_{\text{rel}} = 0.33(V_1' l_1 - V_2' l_2),, \tag{4}$$

where V_i' and l_i are the fluctuation velocity and the path length of particles of the i-th component, respectively.

The formulation of relations describing the kinetic regularities of quasi-diffusive separation is obtained by analyzing the quasi-diffusive particle interaction under two variants of assumptions.

The first assumption [13] is that the fluctuation velocities of the particles are so large that they significantly exceed the mean value of their relative velocity Δu in the shear direction, i.e. $\overline{V'} >> \Delta u$, where $\Delta u = (\partial u / \partial y)\Delta y$. Under this assumption, conditions for the dominance of binary particle collisions are formed in the shear flow, which allows us to express the average value of the particle fluctuation velocity V_i' of the i-th component by the law of conservation of momentum. Using the average values of physical and mechanical characteristics and kinematic parameters of particles in the volume of the elementary layer, the following relation is obtained:

$$V_i' = \overline{V'} \frac{\overline{m}(c)}{m_i}, \tag{5}$$

where $\overline{m}(c)$ is average mass of particles in the volume of the layer. The average particle velocity fluctuations $\overline{V'}$ in the volume of the elementary layer is determined by multiplication of the average values of the particle collision frequency $\overline{F}$ and the distance *s* between them in a hypothetically uniform medium

$$\overline{V'} = \overline{F}\, s\,. \tag{6}$$

The collision frequency $\overline{F}$ is calculated based on the law of conservation of energy, which is generated during shear deformation and dissipated by dissipation effects during particle collision, using the combined oblique impact hypothesis [7].

In accordance with assumption 1 and taking into account dependences (5) and (6), the average amplitude of chaotic displacements of a particle of *i*-type under specific conditions of its contact is determined by the following expression:

$$l_i = s\frac{V_i'}{\overline{V}}\left(\frac{\overline{d}}{d_i}\right)^2 = \frac{\overline{V'}}{\overline{F}}\frac{\overline{m}(c)}{m_i}\left(\frac{\overline{d}}{d_i}\right)^2, \tag{7}$$

where $\overline{d} = cd_1 + (1-c)d_2$ is mean value of particle diameter.

The relative rate of diffusive transport of nonuniform particles, taking into account expressions (4) – (7), can be expressed in the following form

$$q_{\mathrm{rel}} = \frac{\left(\overline{m}(c)\overline{V'}\right)^2}{2\overline{F}}\left(\left(\frac{\overline{d}}{m_1 d_1}\right)^2 - \left(\frac{\overline{d}}{m_2 d_2}\right)^2\right). \tag{8}$$

The change in the concentration of particles of the *i*-th type in the elementary layer of thickness Δy under $c(y) = \mathrm{const}$ will be determined only by the spatial inhomogeneity of the conditions directly affecting the direction and intensity of the transverse quasi-diffusive transport of particles in the shear flow. Since these conditions are fully determined by the nonuniformity of the microstructure of the medium, and according to (4) the most relevant characteristic of the microstructure is the distance between the particles, it is reasonable to express the driving force of quasi-diffusive separation as a relative value of the gradient of the distance between the particles in the *y* direction of quasi-diffusion.

Using an appropriate partial derivative, the driving force of quasi-diffusive separation is formulated as [13]:

$$\frac{\Delta s}{s\Delta y} \cong \frac{1}{s}\frac{\partial s}{\partial y} = \frac{\partial \ln s}{\partial y}. \tag{9}$$

Since the right part of expression (8) has a physical meaning of the quasi-diffusive separation coefficient (migration coefficient [13]) D_m, the value of its flux per unit surface area perpendicular to the separation direction can be expressed in the following form

$$q(m) = \mathrm{c}\rho\frac{\left(\overline{m}(c)\overline{V'}\right)^2}{2\overline{F}}\left(\left(\frac{\overline{d}}{m_1 d_1}\right)^2 - \left(\frac{\overline{d}}{m_2 d_2}\right)^2\right)\frac{\partial \ln \mathrm{s}}{\partial y} =$$

$$= \mathrm{c}\rho D_m\frac{\partial \ln \mathrm{s}}{\partial y}\,. \tag{10}$$

The second assumption concerning the determination of the conditions of quasi-diffusive interaction of particles assumes that the relative velocity of particles in the shear direction essentially exceeds the velocity of their fluctuations $\Delta u >> \overline{V'}$, i.e., under this assumption, the collision of particles occurs with a frequency that depends on their relative velocity, particle properties, and the distance between them. In this case, the dependence of the frequency of collision of particles on the velocity of their fluctuations is negligibly small. As a consequence, the collision frequency is in direct dependence on the relative velocity of particles Δu and is proportional to d^2, and the free path length of particles negligibly little depends on the velocity of their fluctuations. Thus, according to the second variant of the assumption, the free path length is the same for all particles of the mixture and is equal to the average distance between them.

Taking into account the peculiarities of this assumption, the particle fluctuation velocity V_i' of the *i*-th component is determined on the basis of the law of conservation of momentum as a function of the averaged value of the fluctuation velocity of particles of the mixture under constrained conditions of particle collision, in which the total momentum acting on a particle is proportional to its surface and inversely proportional to its mass:

$$V_i' = \overline{m}(c)\overline{V'}m_i^{-1}(d_i/\overline{d})^2\,. \tag{11}$$

According to this assumption, the amplitude of fluctuations is invariant for all particles of the mixture and corresponds to the average value of the distance between them

$$l_i = \overline{l} = s. \tag{12}$$

Taking into account relations (11) and (12), the kinetic coefficient of quasi-diffusive separation caused by mutual quasi-diffusive transfer of nonuniform particles in the shear flow of granular medium under conditions of its structural nonuniformity is expressed [13] in the following form

$$D_m = \frac{\bar{m}(c)(\overline{V'})^2}{2\bar{F}}\left(\frac{d_1^2}{m_1\bar{d}^2} - \frac{d_2^2}{m_2\bar{d}^2}\right). \qquad (13)$$

Obviously, regardless of the version of assumption, the flow intensity of quasi-diffusive separation by size and density in a fast shear flow of particles is determined according to the kinetic dependence (10). The transition from one assumption to another one is carried out by changing the dependence to calculate the process kinetic coefficient. When using the first variant of the assumption, the separation coefficient is calculated according to expression (8), and the second assumption is realized when determining the separation coefficient using the calculation dependence (13).

4.3. Modeling the dynamics of size and density separation under conditions of structural nonuniformity of fast gravitational particle flows

The results of the analysis of the mechanism and kinetics of quasi-diffusive separation were used in mathematical modeling of the dynamics of separation by size and density in a fast gravitational flow of particle mixtures (Table 1) on a rough chute. The modeling was performed using the general mass transfer equation [21], transformed to the specifics of the problem to be solved. In [5, 6, 10, 12, 22], the basic mass transfer equation was adapted to predict the evolution of the particle concentration field of a test component in a gravity flow. In the adapted version, the equation describes the concentration field of particles of the test component with reference to its fluxes of convection, quasi-diffusive mixing and separation in accordance with one of the segregation mechanisms due to the local nonuniformity of the granular medium under gravity flow conditions. According to the results of the analysis, when modeling the separation in gravity flows with high structural nonuniformity, a term is introduced into the equation describing the dynamics of the process, given the kinetics of quasi-diffusive separation (10), which is a consequence of the spatial nonuniformity of the structural characteristics of the flow. After the corresponding addition, the equation of separation dynamics [7, 13, 18] is obtained in the following form

$$\frac{\partial(c\rho)}{\partial t} = -\frac{\partial(uc\rho)}{\partial x} +$$

$$+\frac{\partial}{\partial y}\left[\rho\left(D_{\text{dif}}\frac{\partial c}{\partial y} - cD_m\frac{\partial \ln s}{\partial y} - K_S c\Delta M\right)\right], \qquad (14)$$

where $D_{\text{dif}} = 1/3s\overline{V'}$ is quasi-diffusion mixing coefficient [7, 18].

When modeling the dynamics of the separation process in a gravitational particle flow, equation (14) is integrated numerically using the Crank-Nicholson finite-difference scheme [23]. When solving the equation, the initial condition is set as corresponding to a homogeneous distribution of mixture components in the volume of the bed $c(0, x, y) = c_0$, where c_0 is the average concentration of particles of the test component in the mixture. The boundary conditions are set in the form of relations governing the homogeneous distribution of mixture components on entering the bed ($c(t, 0, y) = c_0$) and the absence of particle flows through the free surface ($y = h$) and the bottom of the bed ($y = 0$)

$$D_{\text{dif}}\frac{\partial c}{\partial y} = cD_m\frac{\partial \ln s}{\partial y} = K_s c\Delta M\big|_{y=0,h} = 0. \qquad (15)$$

A preliminary evaluation of the predictive properties of equation (14) allows us to note that the kinetic coefficients of the quasi-diffusion effects of separation D_m and mixing D_{dif}, as well as the driving force of shear flow segregation ΔM, are analytically determined. The only kinetic parameter in the equation that requires experimental determination is the segregation kinetic coefficient K_S. According to the method described in [19], the segregation coefficient K_S is determined experimentally as the velocity of the test particle during its transverse motion in the shear flow, referred to the driving segregation force ΔM. Approbation of the method has shown that the K_S coefficient obtained using it exhibits the properties of a kinetic constant with respect to a certain mixture of particles in a sufficiently wide range of their properties and structural and kinematic characteristics of the flow [18, 19]. The analysis of equation (14) indicates its sufficiently high predictive capabilities, allowing us to model the dynamics of separation by size and density using only one experimental coefficient.

When modeling the $c(y)$ concentration profiles of test particles in mixtures 1–3 (Table 1) on a rough chute, experimental information on the structural and kinematic characteristics of the flows, obtained as profiles of the void fraction $\varepsilon(y)$ and velocity in the shear direction $u(y)$, was used as input data. The experimental profiles $u(y)$ and $\varepsilon(y)$, as well as the profiles of the concentration of test particles $c(y)$, necessary to evaluate the adequacy of the modeling results, were obtained using the method and experimental unit described in the section "Research methods and model materials".

Figures 5–7 show the results of different variants of mathematical modeling of profiles of concentration distribution profiles of test particles in binary mixtures 1–3 (Table 1) in comparison with experimentally obtained profiles. Mathematical modeling was carried out at different variants of determination of the quasi-diffusion separation coefficient in order to analyze the predictive properties of variants and the significance of the corresponding flux in the separation process.

The analysis of modeling options (Fig. 5) shows that the calculation of the quasi-diffusive separation coefficient using assumption 1 $(\overline{V}' >> \Delta u)$ in accordance with dependence (8) does not provide adequate modeling of the concentration distribution profile.

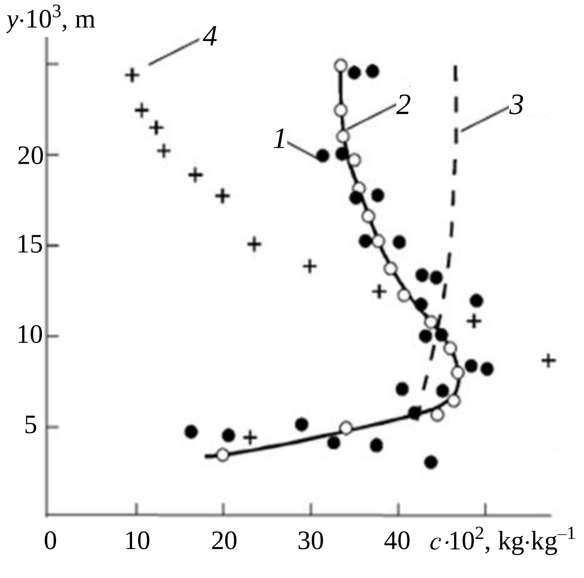

Fig. 5. Concentration profiles of large particles in the flow of mixture *1* (Table 1) on a rough gravity chute, obtained by experimental *1* and analytical *2–4* methods, under different variants of determining the quasi-diffusion separation factor D_m: 2) calculation under assumption (2) in accordance with (13); 3) calculation under $D_m = 0$; 4) calculation under assumption (1) in accordance with (8)

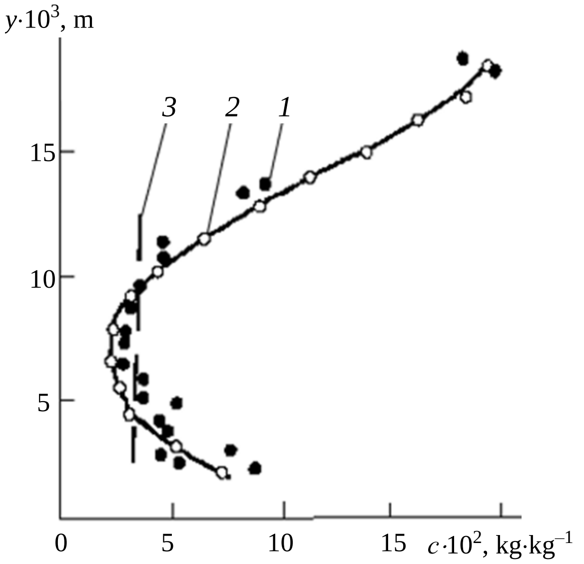

Fig. 6. Experimental *1* and calculated *2* and *3* profiles of concentration distribution of low density particles in the flow of mixture 2 (Table 1) on a rough gravity chute at different variants of determination of quasi-diffusion separation coefficient: 2) calculation under assumption (2) in accordance with (13); 3) calculation under $D_m = 0$

The inadequacy of the modeling option under assumption 1 is also revealed when modeling the separation dynamics in mixtures 2 and 3 in terms of density and complex particle size and density differences.

On the contrary, the use of the second variant of the assumption in modeling, based on the assumption of the dominant role in the process of impact interaction of particles of their relative shear velocity $(\Delta u >> \overline{V}')$, provides adequate results. This is clearly confirmed by comparing the calculated concentration profiles obtained using the second assumption for all types of mixtures (curves *2* in Figs 5–7) with the experimentally obtained concentration distributions of test particles. The adequacy of the calculated results was confirmed at a significance level of 5 % using Fisher's F-criterion to evaluate the ratio of adequacy and reproducibility dispersions.

As a result of analyzing the set of experimental and calculated concentration profiles of cohesionless particles in thin-bed gravity flows (Figs. 5–7), it is possible to conclude that under the influence of structural nonuniformity of flows the effect of quasi-diffusive separation by size and density, which significantly affects the distribution of nonuniform particles, is manifested. This conclusion is confirmed by comparing the calculated concentration profiles obtained considering the quasi-diffusive separation

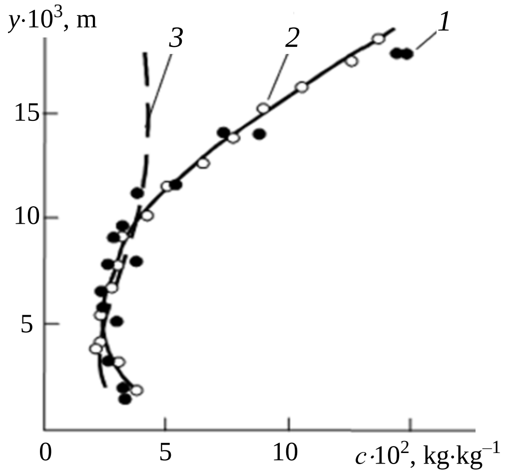

Fig. 7. Experimental *1* and calculated *2* and *3* profiles of concentration distribution of particles with small mass in the flow of mixture 3 (Table 1) on a rough gravity chute at different variants of determination of the quasi-diffusion separation coefficient: 2) calculation under assumption (2) according to (13); 3) calculation at $D_m = 0$

effect (curves *2*) and in the absence of this effect ($D_m = 0$) (curves *3*). In this case, a quasi-diffusive segregation flow is formed due to spatial inhomogeneity of structural and kinematic parameters of the flow, which can dominate over the flow of shear flow segregation, which is a consequence of relaxation of local stress concentrators.

To evaluate the predictive properties of the mathematical model of separation based on equation (14), it is important to note that the concentration profiles of test particles for mixtures 2 and 3, which differ in the disperse composition of components of different densities, are obtained at an invariant value of the kinetic segregation coefficient K_S. Thus, the mathematical model provides the possibility of predicting the distribution of nonuniform in size and density cohesionless spherical particles in gravity flow with high inhomogeneity of structural and kinematic characteristics using a single experimental kinetic parameter.

The determining role of quasi-diffusive separation in accordance with its mechanism under the second variant of the assumption ($\Delta u >> \overline{V}'$) is confirmed by a large number of studies, e.g. [7, 13, 18], performed with mixtures of particles differing in a complex of properties in fast gravity flows in a wide range of flow parameters. However, at the same time, individual cases have been noted, indicating that in special flow conditions of a certain type of granular matter it is reasonable to describe the kinetics of quasi-diffusive separation using the first variant of the assumption.

Figure 8 shows the calculated profiles (curves *2*(1) and *2*(2)) of the concentration distribution of small particles in the mixture of glass bead fractions in a fast gravity flow. When modeling profile *2*(1) in the upper rarefied zone of the flow ($\varepsilon \geq 0.75$), the quasi-diffusive separation coefficient D_m is calculated by (8) (according to the first version of the assumption) and by (13) (according to the second version of the assumption) in the rest of the flow volume. In the modeling of profile 2(2), the quasi-diffusive separation coefficient D_m is calculated by (13) (according to the second version of the assumption) in the whole flow volume. The simulation results (curves *2*(1) and *2*(2)) are presented in comparison with the experimental concentration distribution profile (curve *1*). Comparison of modeling variants (Fig. 8) shows that within the analyzed object adequate modeling of quasi-diffusive separation in a rarefied flow region at the void fraction above 0.75 is provided using the hypothesis of fluctuation velocity ($\overline{V}' >> \Delta u$)dominance in relative particle movements according to the first variant of the assumption. This can be explained by presumably very low effects of dissipation of kinetic energy at collision of glass beads particles (high restitution coefficient and low friction coefficient) in the rarefied region of the shear flow "cloud" at low gradient of particle velocity in the shear direction.

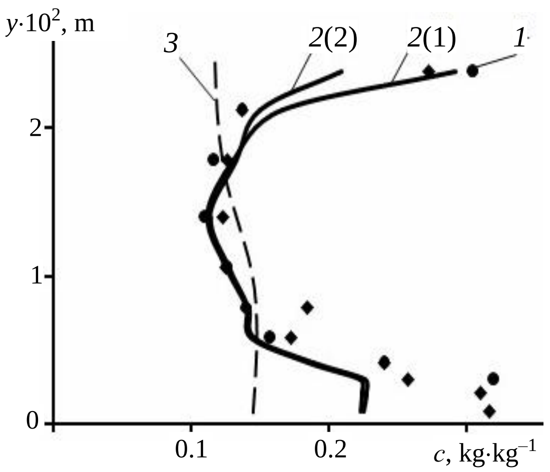

Fig. 8. Concentration profiles of glass beads particles of fine fraction (+2.5–3.0) mm in mixture with coarse fraction (+3.25–3.5) mm under gravitational flow on rough chute: *1* – experimental; *2*, *3* – calculated at different variants of determination of quasi-diffusion separation coefficient D_m : *2*(1) calculation at the first variant of assumption according to (8); *2*(2) calculation at the second variant of assumption according to (13); 3) calculation at $D_m = 0$

Obviously, under the action of turbulent pulsations occurring in the flow core and at low intensity of the effects of dissipation of kinetic energy in binary particle collisions in the flow "cloud", the conditions corresponding to the first variant of the assumption ($\overline{V}' >> \Delta u$)can be fulfilled.

The analyzed mechanism of quasi-diffusion separation enables to explain the physical nature of some paradoxical effects that were found in the experimental study of failure zones formed in the process of shear deformations of mixtures of granular matter in the annular shear cell and can be taken into account when organizing the processes of preparing mixtures of composite materials and their structuring [24, 25]. For example, according to the authors of [12], the mysterious behavior of mustard seeds in the shear flow of their mixture with bead granules is probably explained by the formation of shear flows with large gradients of shear strain rate and, accordingly, the fraction of voids in the flow volume between its core and the cell walls. Under conditions of void fraction gradients, mutual movement of mustard seeds and glass particles will occur under the effect of quasi-diffusive separation. Less massive mustard seeds will move in the direction of the void fraction gradient toward the cell walls while heavy glass particles will move toward the center of the failure zone. Due to the quasi-diffusion effect, mustard seeds are distributed in the near-wall zones of the cell with replacement of the denser glass particles into the core of the shear flow. For a more convincing explanation of the extraordinary separation effects, detailed experimental information on the structural characteristics in the shear flow volume in the annular cell is required.

5. Conclusion

The physical nature and mechanisms of quasi-diffusive separation of cohesionless spherical particles with a complex difference in size and density in fast shear flows with high inhomogeneity of structural and kinematic parameters have been analyzed. It is established that in the conditions of thin-bed fast gravitational flows on a rough chute the determining role in the formation of quasi-diffusive separation flow belongs to the character of structural nonuniformity of the flow. The analysis of kinetic regularities of the quasi-diffusive separation process in alternative conditions of quasi-diffusive interaction of particles in a fast shear flow, which are characterized by the dominance of either the velocity of chaotic fluctuations of particles or their relative velocity in the shear direction.

The results of experimental and analytical study show that the intensity of quasi-diffusive separation flow is in direct dependence on the particle collision frequency. It is established that in the general case of steady fast gravitational flow of granular matter on a rough chute, the collision frequency is determined by the dominant value of the component of the relative shear velocity of particles and depends to a lesser extent on the velocity of their chaotic fluctuations. At the same time, under conditions of fast gravitational flow of smooth elastic spherical particles in the region of the "cloud" of particles near the open surface of the flow, the frequency of particle collisions can be determined at the dominant value of the velocity of their fluctuations. It has been established that in thin-bed fast gravitational flows quasi-diffusive separation due to spatial structural nonuniformity of the flow can dominate over segregation as a manifestation of the effect of local nonuniformity of the shear flow.

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7. Conflict of interests

The authors declare no conflict of interests.

References

1. Nagel SR. Experimental soft-matter science. *Reviews of Modern Physics*. 2017;89(2):025002. DOI:10.1103/RevModPhys.89.025002
2. Forterre Y, Pouliquen O. Flows of dense granular media. *Annual Review of Fluid Mechanics*. 2008;40(1):1-24. DOI:10.1146/annurev.fluid.40.111406.102142
3. Shen HH, Ackermann NL. Constitutive relationships for fluid-solid mixtures. *Journal of Engineering Mechanics-ASCE*. 1982;108(5):748-763. DOI:10.1061/JMCEA3.0002868
4. Campbell CS. Granular material flows – an overview. *Powder Technology*. 2006;162(3):208-229. DOI:10.1016/j.powtec.2005.12.008
5. Duan Y, Peckham J, Umbanhowar PB, Ottino JM, et al. Designing minimally segregating granular mixtures for gravity-driven surface flows. *AIChE Journal*. 2023;69(4): e18032. DOI:10.1002/aic.18032
6. Duan Y, Umbanhowar PB, Ottino JM, Lueptow RM. Modelling segregation of bidisperse granular mixtures varying simultaneously in size and density for free surface flows. *Journal of Fluid Mechanics*. 2021;918:A20. DOI:10.1017/jfm.2021.342
7. Dolgunin VN, Kudi AN, Tuev MA. Mechanisms and kinetics of gravity separation of granular materials. *Physics-Uspekhi*. 2020;63(6):545-561. DOI:10.3367/UFNe.2020.01.038729
8. Hill KM, Fan Y. Granular temperature and segregation in dense sheared particulate mixtures. *KONA Powder and Particle Journal*. 2016;33:150-168. DOI:10.14356/kona.2016022

9. Savage SB. Granular flows down rough inclines – review and extension. In: *Studies in Applied Mechanics*. Elsevier; Vol. 7, 1983. p. 261-282.

10. Gray JMNT, Thornton AR. A theory for particle size segregation in shallow granular free-surface flows. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*. 2005;461(2057):1447-1473. DOI:10.1098/rspa.2004.1420

11. Goldhirsch I. Rapid granular flows. *Annual Review of Fluid Mechanics*. 2003;35:267-293. DOI:10.1146/annurev.fluid.35.101101.161114

12. Stephens DJ, Bridgwater J. The mixing and segregation of cohesionless particulate materials. Part I. Failure zone formation. *Powder Technology*. 1978;21(1):17-28. DOI:10.1016/0032-5910(78)80104-1

13. Stephens DJ, Bridgwater J. The mixing and segregation of cohesionless particulate materials. Part II. Microscopic mechanisms for particles differing in size. *Powder Technology*. 1978;21(1):29-44. DOI:10.1016/0032-5910(78)80105-3

14. Dolgunin VN, Kudy AN, Ukolov AA. Development of the model of segregation of particles undergoing granular flow down an inclined chute. *Powder Technology*. 1998;96(3):211-218. DOI:10.1016/S0032-5910(97)03376-7

15. Bagnold RA. Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*. 1954;225(1160):49-63. DOI:10.1098/rspa.1954.0186

16. Khakhar DV, McCarthy JJ, Ottino JM. Radial segregation of granular mixtures in rotating cylinders. *Physics of Fluids*. 1997;9(12):3600-3614. DOI:10.1063/1.869498

17. Hill KM, Tan DS. Segregation in dense sheared flows: gravity, temperature gradients, and stress partitioning. *Journal of Fluid Mechanics*. 2014;756:5488. DOI:10.1017/jfm.2014.271

18. Gray JMNT, Chugunov VA. Particle-size segregation and diffusive remixing in shallow granular avalanches. *Journal of Fluid Mechanics*. 2006;569:365. DOI:10.1017/S0022112006002977

19. Dolgunin VN, Kudi AN, Tarakanov AG. Structural inhomogeneity and effects of separation by size and density in gravity flow of granular materials. *Journal of Engineering Physics and Thermophysics*. 2022;95(2):484-494. DOI:10.1007/s10891-022-02505-y

20. Dolgunin VN, Ukolov AA, Ivanov OO. Segregation kinetics in the rapid gravity flow of granular materials. *Theoretical Foundations of Chemical Engineering*. 2006;40(4):393-404. DOI:10.1134/S0040579506040099

21. Ferziger JH, Kaper HG. *Mathematical theory of transport processes in gases*. Amsterdam: North-Holland Publ. house; 1972. 606 p.

22. Lykov AV. *Heat and mass transfer: reference book*. Moscow: Energy; 1978. 480 p. (In Russ.)

23. Gray JMNT. Particle segregation in dense granular flows. *Annual Review of Fluid Mechanics*. 2018;50(1):407-433. DOI:10.1146/annurev-fluid-122316-045201

24. Marchuk GI. *Methods of numerical mathematics*. New York: Springer; 1975. 510 p.

25. Klyuev AV, Kashapov NF, Klyuev SV, Zolatareva SV, et al. Experimental studies of the processes of structure formation of composite mixtures with technogenic mechanoactivated silica component. *Stroitel'nye Materialy i Izdeliya = Construction Materials and Products*. 2023;6(2):5-18. DOI:10.58224/2618-7183-2023-6-2-5-18 (In Russ.)

26. Klyuev AV, Kashapov NF, Klyuev SV, Lesovik RV, et al. Development of alkali-activated binders based on technogenic fibrous materials. *Stroitel'mye Materialy i Izdeliya = Construction Materials and Products*. 2023;6(1):60-73. DOI:10.58224/2618-7183-2023-6-1-60-73 (In Russ.)

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