Том 24, № 128

© Grosheva L.I., 2019 DOI 10.20310/2686-9667-2019-24-128-368-375 УДК 517.98

## Decomposition of boundary representations on the Lobachevsky plane associated with linear bundles

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# Разложение граничных представлений на плоскости Лобачевского в сечениях линейных расслоений

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Abstract. Earlier we described canonical (labelled by  $\lambda \in \mathbb{C}$ ) and accompanying boundary representations of the group  $G = \mathrm{SU}(1,1)$  on the Lobachevsky plane D in sections of linear bundles and decomposed canonical representations into irreducible ones. Now we decompose representations acting on distributions concentrated at the boundary of D. In the generic case  $2\lambda \notin \mathbb{N}$  they are diagonalizable, in the exceptional case Jordan blocks appear.

**Keywords:** Lobachevsky plane; canonical representations; distributions; boundary representations; Poisson transforms

For citation: Grosheva L.I. Razlozhenie granichnyh predstavlenij na ploskosti Lobachevskogo v secheniyah linejnyh rassloenij [Decomposition of boundary representations on the Lobachevsky plane associated with linear bundles]. Vestnik rossiyskikh universitetov. Matematika – Russian Universities Reports. Mathematics, 2019, vol. 24, no. 128, pp. 368–375. DOI 10.20310/2686-9667-2019-24-128-368-375.

Аннотация. Ранее мы описали канонические и граничные представления группы G = SU(1, 1) на плоскости Лобачевского в сечениях линейных расслоений (они нумеруются комплексными числами  $\lambda$ ) и разложили канонические представления на неприводимые. Сейчас мы разлагаем представления, действующие в обобщенных функциях, сосредоточенных на границе. В общем случае  $2\lambda \notin \mathbb{N}$  они диагонализуемы, в исключительном случае появляются жордановы клетки.

**Ключевые слова:** плоскость Лобачевского; канонические представления; обобщенные функции; граничные представления; преобразования Пуассона

Для цитирования: Грошева Л.И. Разложение граничных представлений на плоскости Лобачевского в сечениях линейных расслоений // Вестник российских университетов. Математика. 2019. Т. 24. № 128. С. 368–375. DOI 10.20310/2686-9667-2019-24-128-368-375. (In Engl., Abstr. in Russian)

In our work [3] we described canonical and boundary representations of the group G = SU(1, 1) on the Lobachevsky plane D in sections of linear bundles on D. Then in [4] we decomposed *canonical* representations into irreducible ones. Now we continue [4] and decompose *boundary* representations. We lean on works [1,2].

## 1. The Lobachevsky plane

The Lobachevsky plane is the unit disk  $D : z\overline{z} < 1$  on the complex plane with the linear-fractional action of G:

$$z \mapsto z \cdot g = \frac{az + \overline{b}}{bz + \overline{a}}, \quad g = \left(\begin{array}{cc} a & b \\ \overline{b} & \overline{a} \end{array}\right), \quad a\overline{a} - b\overline{b} = 1.$$

The boundary S of D is the circle  $z\overline{z} = 1$ , it consists of points  $s = \exp i\alpha$ , the measure ds on S is  $d\alpha$ . Let  $\overline{D}$  be the closure of  $D: \overline{D} = D \cup S$ . Let

$$p = 1 - z\overline{z},$$

so that  $D = \{p > 0\}$  and  $S = \{p = 0\}$ . The stabilizer of the point z = 0 is the maximal compact subgroup K = U(1) consisting of diagonal matrices:

$$k = \left(\begin{array}{cc} a & 0\\ 0 & \overline{a} \end{array}\right) , \quad a\overline{a} = 1,$$

so that D = G/K. The Euclidean measure dxdy on D is (1/2) dp ds, a G-invariant measure  $d\mu(z)$  on D is

$$d\mu(z) = p^{-2} dx dy.$$

If M is a manifold, then  $\mathcal{D}(M)$  denotes the space of compactly supported infinitely differentiable  $\mathbb{C}$ -valued functions on M, with a usual topology, and  $\mathcal{D}'(M)$  denotes the space of distributions on M – of antilinear continuous functionals on  $\mathcal{D}(M)$ .

We use the notation

$$\mathbb{N} = \{0, 1, 2, \ldots\}.$$

Recall principal non-unitary series representations of G trivial on the center, see also [4]. Let  $\sigma \in \mathbb{C}$ . The representation  $T_{\sigma}$  acts on the space  $\mathcal{D}(S)$  by

$$(T_{\sigma}(g)\varphi)(s) = \varphi(s \cdot g)|bs + \overline{a}|^{2\sigma}$$

The inner product from  $L^2(S, ds)$ :

$$\langle \psi, \varphi \rangle_S = \int_S \psi(s) \,\overline{\varphi(s)} \, ds$$

is invariant with respect to the pair  $(T_{\sigma}, T_{-\overline{\sigma}-1})$ .

If  $\sigma \notin \mathbb{Z}$ , then  $T_{\sigma}$  is irreducible and equivalent to  $T_{-\sigma-1}$  (for  $\sigma \in \mathbb{Z}$  there is a "partial equivalence"). For  $\sigma = v \in \mathbb{N}$  the representation  $T_v$  has an invariant irreducible subspace  $E_v$  spanned by  $\exp ir\alpha$ ,  $r = -v, -v + 1, \dots, v$ .

A basis of the Lie algebra  $\mathfrak{g}$  of G is

$$L_0 = \begin{pmatrix} i/2 & 0 \\ 0 & -i/2 \end{pmatrix}, \quad L_1 = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}.$$

We also use their linear combinations (they belong to the complexification of  $\mathfrak{g}$ ):

$$L_{+} = L_{2} + iL_{1} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad L_{-} = L_{2} - iL_{1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Denote by  $\Delta_{\mathfrak{g}}$  the twice Casimir element of the universal enveloping algebra  $\operatorname{Env}(\mathfrak{g})$  of  $\mathfrak{g}$ :

$$\Delta_{\mathfrak{g}} = -L_0^2 + L_1^2 + L_2^2.$$

The representation  $T_{\sigma}$  of  $\mathfrak{g}$  assigns to  $L_0$ ,  $L_+$ ,  $L_-$  the following operators:

$$T_{\sigma}(L_0) = \frac{d}{d\alpha},$$
  

$$T_{\sigma}(L_+) = e^{i\alpha} \left(\sigma + i\frac{d}{d\alpha}\right),$$
  

$$T_{\sigma}(L_-) = e^{-i\alpha} \left(\sigma - i\frac{d}{d\alpha}\right),$$
  

$$T_{\sigma}(\Delta_{\mathfrak{g}}) = \sigma(\sigma + 1).$$

## 2. Canonical representations

Let  $\mathcal{D}(\overline{D})$  be the space of restrictions to  $\overline{D}$  of functions from  $\mathcal{D}(\mathbb{C})$  with the induced topology, and by  $\mathcal{D}'(\overline{D})$  the space of distributions on  $\mathbb{C}$  with supports in  $\overline{D}$ . Consider the inner product with respect to the Lebesgue measure on D:

$$\langle F, f \rangle_D = \int_D F(z)\overline{f(z)}dxdy, \quad z = x + iy.$$
 (2.1)

The space  $\mathcal{D}(\overline{D})$  can be embedded into  $\mathcal{D}'(\overline{D})$  by assigning to  $h \in \mathcal{D}(\overline{D})$  the functional  $f \mapsto \langle h, f \rangle_D, f \in \mathcal{D}(\overline{D}).$ 

We shall use denotation:

$$z^{\mu,m} = |z|^{\mu} \left(\frac{z}{|z|}\right)^m, \quad \mu \in \mathbb{C}, \quad m \in \mathbb{Z}.$$

Let  $\lambda \in \mathbb{C}$ . We define the *canonical representation*  $R_{\lambda,m}$  of the group G associated with a character of K as follows:

$$(R_{\lambda,m}(g)f)(z) = f(z \cdot g) (bz + \overline{a})^{-2\lambda - 4,2m},$$

it acts on the space  $\mathcal{D}(\overline{D})$ .

The inner product (2.1) is invariant with respect to the pair  $(R_{\lambda,m}, R_{-\overline{\lambda}-2,m})$ :

$$\langle R_{\lambda,m}(g)f, h \rangle_D = \langle f, R_{-\overline{\lambda}-2,m}(g^{-1})h \rangle_D, g \in G.$$
 (2.2)

The formula (2.2) allows to extend the representation  $R_{\lambda,m}$  to the space  $\mathcal{D}'(\overline{D})$  of distributions on  $\overline{D}$ .

Here are formulae for basic elements of g in variables p and  $\alpha$ :

$$R_{\lambda,m}(L_0) = \frac{\partial}{\partial \alpha} - im,$$
  
$$R_{\lambda,m}(L_{\pm}) = e^{\pm i\alpha} \left\{ -rp \frac{\partial}{\partial p} \pm \frac{1}{2} (r + r^{-1}) i \frac{\partial}{\partial \alpha} - (\lambda + 2 \mp m) r \right\}.$$
 (2.3)

.

Let us also write the operator corresponding to  $\Delta_{\mathfrak{g}}$ :

$$R_{\lambda,m}(\Delta_{\mathfrak{g}}) = (p^3 - p^2)\frac{\partial^2}{\partial p^2} + \left[(2\lambda + 4)p - (2\lambda + 5)p^2\right]\frac{\partial}{\partial p} + imp \ \frac{\partial}{\partial \alpha} + \frac{1}{4} \cdot \frac{p^2}{1 - p} \ \frac{\partial^2}{\partial \alpha^2} +$$

+ 
$$[(\lambda + 2)(\lambda + 1) - ((\lambda + 2)^2 - m^2)p].$$
 (2.4)

In (2.4) one has to use the binomial expansions ( $r = (1 - p)^{1/2}$ ):

$$r = \sum_{n=0}^{\infty} {\binom{1/2}{n}} (-1)^n p^n, \qquad (2.5)$$

$$r^{-1} = \sum_{n=0}^{\infty} {\binom{-1/2}{n}} (-1)^n p^n, \qquad (2.6)$$

$$\frac{1}{2}(r+r^{-1}) = \sum_{n=0}^{\infty} {\binom{1/2}{n}} (1-n)(-1)^n p^n.$$
(2.7)

Applying these formulae to distributions  $\zeta$ , we have to keep in mind the following:

$$p^n \delta^{(k)}(p) = (-1)^n \frac{k!}{(k-n)!} \delta^{(k-n)}(p).$$

## 3. Boundary representations

Canonical representations  $R_{\lambda,m}$  generate two boundary representations  $L_{\lambda,m}$  and  $M_{\lambda,m}$ . For simplicity, in this paper we restrict ourselves to the first one. It acts on distributions in  $\mathcal{D}'(D)$  concentrated at S.

Consider distributions of the following form:

$$\zeta = \varphi(s)\,\delta^{(k)}(p),$$

where  $\varphi \in \mathcal{D}(S)$  and  $\delta(p)$  is the Dirac delta function on the real line (being a continuous linear functional on  $\mathcal{D}(\mathbb{R})$ ) and  $\delta^{(k)}(p)$  its k-th derivative. The space of these distributions will be denoted by  $\Delta_k(\overline{D})$ . Define also

$$\Sigma_k(\overline{D}) = \Delta_0(\overline{D}) + \Delta_1(\overline{D}) + \dots + \Delta_k(\overline{D}),$$

so that a distribution  $\zeta$  in  $\Sigma_k(\overline{D})$  is a linear combination

$$\zeta = \varphi_0(s)\,\delta(p) + \varphi_1(s)\,\delta'(p) + \dots + \varphi_k(s)\,\delta^{(k)}(p).$$

We get a filtration:

$$\Delta_0(\overline{D}) = \Sigma_0(\overline{D}) \subset \Sigma_1(\overline{D}) \subset \Sigma_2(\overline{D}) \subset \dots$$

Let  $\Sigma(\overline{D})$  denote the union of all  $\Sigma_k(\overline{D})$ .

The canonical representation  $R_{\lambda,m}$  acting on  $\mathcal{D}'(\overline{D})$ , preserves the space  $\Sigma(\overline{D})$  and the filtration (2.3). The boundary representation  $L_{\lambda,m}$  is the restriction of  $R_{\lambda,m}$  to  $\Sigma(\overline{D})$ .

## 4. Poisson transform

Let  $\lambda, \sigma \in \mathbb{C}$  and  $m \in \mathbb{Z}$ . We define the Poisson transform associated with the canonical representation  $R_{\lambda,m}$  as the map  $P_{\lambda,\sigma}^{(m)} : \mathcal{D}(S) \to C^{\infty}(D)$  by the following formula

$$\left(P_{\lambda,\sigma}^{(m)}\varphi\right)(z) = p^{-\lambda-\sigma-2} \int_{S} (1-s\overline{z})^{2\sigma,-2m} s^{m} \varphi(s) \, ds.$$

The Poisson transform  $P_{\lambda,\sigma}^{(m)}$  intertwines the representations  $T_{-\sigma-1}$  and the canonical representation  $R_{\lambda,m}$ :

$$R_{\lambda,m}(g) P_{\lambda,\sigma}^{(m)} = P_{\lambda,\sigma}^{(m)} T_{-\sigma-1}(g) \quad (g \in G).$$

The Poisson transform  $P_{\lambda,\sigma}^{(m)}$  is meromorphic in  $\sigma$ , and has poles at the points

$$\sigma = \lambda - k, \quad \sigma = -\lambda - 1 + l \quad (k, l \in \mathbb{N}).$$
(4.1)

All poles are simple except in the case when the two sequences (4.1) have a non-empty intersection and the pole belongs to this intersection. This happens when  $2\lambda + 1 \in \mathbb{N}$  and  $0 \leq k, l \leq 2\lambda + 1, k + l = 2\lambda + 1$ . In this case the pole  $\mu$  is of the second order. Let us write down the principal part of the Laurent series of  $P_{\lambda,\sigma}^{(m)}$  at the poles  $\mu$  of the first order:

$$P_{\lambda,\sigma}^{(m)} = \frac{\widehat{P}_{\lambda,\mu}^{(m)}}{\sigma - \mu} + \cdots$$

The residue intertwines  $T_{-\mu-1}$  with  $R_{\lambda,m}$ . Let us write it explicitly. We set

$$V_{\sigma,m,n}(p) = (1-p)^{(m+n)/2} F(\sigma + 1 + m, \sigma + 1 + n; 2\sigma + 2; p),$$

where F is the Gauss hypergeometric function. Expand V in powers of p:

$$V_{\sigma,m,n}(p) = \sum_{k=0}^{\infty} w_{\sigma,k}^{(m)}(n) p^k,$$

here  $w_{\sigma,k}^{(m)}$  are polynomials in *n* of degree *k*. The coefficients of these polynomials are rational functions of  $\sigma$  with simple poles. We set

$$W_{\sigma,k}^{(m)} = w_{\sigma,k}^{(m)} \left(\frac{1}{i} \frac{d}{d\alpha}\right).$$

If a pole  $\mu$  belongs only to one of the sequences (4.1), then it is simple. In particular,

$$\widehat{P}_{\lambda,\lambda-k}^{(m)} = (-1)^{k+m} \frac{1}{k!} a_{-m}(\lambda-k) \,\xi_{\lambda,k}^{(m)},$$

where

$$a_n(\sigma) = 2\pi \, (-1)^n \, \frac{\Gamma(-2\sigma - 1)}{\Gamma(-\sigma + n) \, \Gamma(-\sigma - n)}$$

and  $\xi_{\lambda,k}^{(m)}$  is the following operator  $\mathcal{D}(S) \to \Sigma_k(\overline{D})$ :

$$\xi_{\lambda,k}^{(m)} \varphi = s^m \sum_{n=0}^k (-1)^n \frac{k!}{(k-n)!} \left( W_{\lambda-k,n}^{(m)} \varphi \right)(s) \, \delta^{(k-n)}(p).$$

The operator  $\xi_{\lambda,k}^{(m)}$  is meromorphic in  $\lambda$ . For fixed k = 1, 2... it has k poles (simple) at the points  $\lambda = k - 1, k - 3/2, k - 2, ..., (k - 1)/2$ . It intertwines  $T_{-\lambda-1+k}$  with  $L_{\lambda,m}$  (restricted to  $\Sigma_k(\overline{D})$ ).

Let us write three first operators:

$$\begin{split} \xi_{\lambda,0}^{(m)} \varphi &= s^m \varphi \cdot \delta(p), \\ \xi_{\lambda,1}^{(m)} \varphi &= s^m \Big\{ \varphi \cdot \delta(p) - \frac{1}{2\lambda} (\lambda^2 \varphi - m \cdot i\varphi') \cdot \delta'(p) \Big\}, \\ \xi_{\lambda,2}^{(m)} \varphi &= s^m \Big\{ \varphi \cdot \delta(p) - \frac{1}{\lambda - 1} \big( (\lambda - 1)^2 \varphi - m \cdot i\varphi' \big) \cdot \delta'(p) \\ &+ \frac{1}{4(\lambda - 1)(2\lambda - 1)} \Big\{ \left[ (\lambda - 1)^2 \lambda^2 + m^2 \right] \varphi \\ &- 2(\lambda - 1)(2\lambda - 1) m \cdot i\varphi' - \left[ (\lambda - 1)^2 + 2m^2 \right] \varphi'' \Big\} \cdot \delta''(p) \Big\}. \end{split}$$

## 5. Decomposition of boundary representations

**Theorem 5.1.** The representation  $L_{\lambda,m}$  is equivalent to a upper triangular matrix with diagonal  $T_{-\lambda-1}, T_{-\lambda}, T_{-\lambda+1}, \ldots$ 

P r o o f. The formulae (2.3) and (2.5)–(2.7) show that operators  $R_{\lambda,m}(L^{\pm})$ move subspaces  $\Delta_k(\overline{D})$  to  $\Sigma_k(\overline{D})$ . Also these formulae show that the operator  $R_{\lambda,m}(X)$ where  $X \in \mathfrak{g}$  moves a distribution  $s^m \varphi(s) \delta^{(k)}(p)$  in  $\Delta_k(\overline{D})$  to the distribution  $s^m(T_{-\lambda-1+k}(X)\varphi)(s) \delta^{(k)}(p) + \dots$  in  $\Sigma_k(\overline{D})$ .

Let  $V_{\lambda,k}^{(m)}$  be the image of  $\xi_{\lambda,k}^{(m)}$ . This space is contained in  $\Sigma_k(\overline{D})$  and its projection to  $\Delta_k(\overline{D})$  is the whole  $\Delta_k(\overline{D})$ . It gives:

**Theorem 5.2.** In the generic case  $2\lambda \notin \mathbb{N}$  the boundary representation  $L_{\lambda,m}$  is diagonalizable which means that  $\Sigma(\overline{D})$  decomposes into the direct sum of subspaces  $V_{\lambda,k}^{(m)}$ ,  $k \in \mathbb{N}$ , the restriction of  $L_{\lambda,m}$  to  $V_{\lambda,k}^{(m)}$  is equivalent to  $T_{-\lambda-1+k}$  (by  $\xi_{\lambda,k}$ ), so that  $L_{\lambda,m}$  is the direct sum of the  $T_{-\lambda-1+k}$  ( $k \in \mathbb{N}$ ).

Now let  $\lambda \in (1/2)\mathbb{N}$ . This number  $\lambda$  is a pole (of the first order) of  $\xi_{\tau,k}^{(m)}$  in  $\tau$  for  $k \in \mathbb{N}$  such that  $\lambda + 1 \leq k \leq 2\lambda + 1$ . For example, if  $\lambda = 0$ , then k = 1; if  $\lambda = 1/2$ , then k = 2; if  $\lambda = 1$ , then k = 2,3; if  $\lambda = 3/2$ , then k = 3,4. For these  $\lambda$  the spaces  $V_{\lambda,k}^{(m)}$  are defined for all  $k \in \mathbb{N}$  such that  $k < \lambda + 1$  and  $2\lambda + 1 < k$ , for the others these spaces are absent. Let us write down the Laurent expansion of  $\xi_{\tau,k}^{(m)}$  at  $\tau = \lambda$ :

$$\xi_{\tau,k}^{(m)} = \frac{\widehat{\xi}_{\lambda,k}^{(m)}}{\tau - \lambda} + \mathring{\xi}_{\lambda,k}^{(m)} + \dots$$

For the indicated k we define the spaces  $\widehat{V}_{\lambda,k}^{(m)}$  and  $\mathring{V}_{\lambda,k}^{(m)}$  as the images of the operators  $\widehat{\xi}_{\lambda,k}^{(m)}$  and  $\mathring{\xi}_{\lambda,k}^{(m)}$  respectively. The space  $\widehat{V}_{\lambda,k}^{(m)}$  is isomorphic to  $V_{\lambda,l}^{(m)}$  where  $l + k = 2\lambda + 1$ , namely there is a relation  $\mathring{\xi}_{\lambda,k}^{(m)}(\varphi) = \xi_{\lambda,l}^{(m)}(\psi)$  where  $\psi$  is obtained from  $\varphi$  by means of some operator. Therefore the operator  $\widehat{\xi}_{\lambda,k}^{(m)}$  intertwines  $T_{-\lambda-1+l}$  with  $L_{\lambda,m}$ , notice that it vanishes on  $E_l$ . The space  $\mathring{V}_{\lambda,k}^{(m)}$  has the full projection to  $\Delta_k(\overline{D})$ .

On the pair  $\widehat{V}_{\lambda,k}^{(m)}$ ,  $\mathring{V}_{\lambda,k}^{(m)}$  the representation  $L_{\lambda,m}$  is the block

$$\left(\begin{array}{cc} T_{-\lambda-1+l} & * \\ 0 & T_{-\lambda-1+k} \end{array}\right).$$

Since  $-\lambda - 1 + l = -(-\lambda - 1 + k) - 1$ , representations  $T_{-\lambda - 1 + l}$  and  $T_{-\lambda - 1 + k}$  are isomorphic, so that this block is a genuine Jordan block. Here is the matrix corresponding to the Casimir operator  $R_{\lambda,m}(\Delta_g)$ :

$$\left(\begin{array}{cc} \mu(\mu+1) & * \\ 0 & \mu(\mu+1) \end{array}\right)$$

where  $\mu = -\lambda - 1 + l$  or  $\mu = -\lambda - 1 + k$ . Thus, me obtain the following theorem (we use the notation [a] for the integral part of a number a).

**Theorem 5.3.** Let  $\lambda \in (1/2)\mathbb{N}$ . Then the space  $\Sigma(\overline{D})$  is the direct sum of the subspaces  $V_{\lambda,k}^{(m)}$  with  $k \ge 2\lambda + 2$  and  $k \le \lambda$  and subspaces  $\mathring{V}_{\lambda,k}^{(m)}$  with  $\lambda + 1 \le k \le 2\lambda + 1$ .

The representation  $L_{\lambda,m}$  is equivalent to the direct sum of  $[\lambda+1]$  Jordan blocks with the diagonal  $(T_{-\lambda-1+j}, T_{\lambda-j})$ ,  $j = 0, 1, \ldots, [\lambda]$ , acting on subspaces  $V_{\lambda,l}^{(m)} + \mathring{V}_{\lambda,k}^{(m)}$ ,  $k+l = 2\lambda+1$ , the representation  $T_{1/2}$  for half-integer  $\lambda$ , and the representations  $T_{\lambda+1}, T_{\lambda+2}, \ldots$ 

Let us write  $\hat{\xi}_{\lambda,k}^{(m)}$  and  $\dot{\xi}_{\lambda,k}^{(m)}$  for some  $\lambda$ , k. Let  $\lambda = 0, \ k = 1$ . Then

$$\widehat{\xi}_{0,1}^{(m)}(\varphi) = \frac{im}{2} s^m \varphi' \delta(p), \quad \mathring{\xi}_{0,1}^{(m)}(\varphi) = s^m \varphi \delta'(p).$$

Let  $\lambda = 1$ , k = 2. Then

$$\begin{aligned} \widehat{\xi}_{1,2}^{(m)}(\varphi) &= ims^m \Big\{ \varphi' \delta'(p) - \frac{1}{2} \left( \varphi' - im\varphi'' \right) \delta(p) \Big\}, \\ \mathring{\xi}_{1,2}^{(m)}(\varphi) &= s^m \Big\{ \varphi \delta''(p) + \frac{1}{4} \left( m^2 - 2im\varphi' + (4m^2 - 1)\varphi'' \right) \delta(p) \Big\}. \end{aligned}$$

Let  $\lambda = 1/2$ , k = 2. Then

$$\begin{split} \widehat{\xi}_{1/2,2}^{(m)}(\varphi) &= \frac{1}{32} (4m^2 - 1) s^m \left(\varphi + 4\varphi''\right) \delta(p), \\ \mathring{\xi}_{1/2,2}^{(m)}(\varphi) &= s^m \Big\{ \varphi \delta''(p) + \frac{1}{2} \left(\varphi - 4im\varphi'\right) \delta'(p) \\ &+ \frac{1}{16} \left(4\varphi + im\varphi' + 16m^2\varphi''\right) \delta(p) \Big\}. \end{split}$$

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Received 13 August 2019 Reviewed 16 October 2019 Accepted for press 29 November 2019

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Поступила в редакцию 13 августа 2019 г. Поступила после рецензирования 16 октября 2019 г. Принята к публикации 29 ноября 2019 г.