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$\rho - F$-contraction fixed point theorem

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Abstract. In this paper, we study the question of conditions for the existence and uniqueness of a fixed point of a mapping over a complete metric space. We first discuss the concepts of F-contraction and F^*-contraction in fixed point theory. These concepts, developed respectively by Wardowski and Piri with Kumam, have catalyzed significant research in various metric spaces. We then propose a generalization of these concepts, $\rho - F$-contraction and $\rho - F^*$-contraction, and demonstrate its effectiveness in ensuring the existence and uniqueness of fixed points. This new approach provides greater flexibility by including a function ρ that modulates the contraction, extending the applicability of F- and F^*-contractions. We conclude the paper with an example of a mapping that is a $\rho - F$-contraction and a $\rho - F^*$-contraction, respectively, and has a unique fixed point. However, this mapping does not satisfy the conditions of Wardowski and the conditions of Piri and Kumam.

Keywords: fixed-point, existence, uniqueness, F-contraction, $\rho - F$-contraction

Mathematics Subject Classification: 47H10, 54E35.

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НАУЧНАЯ СТАТЬЯ

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Теорема о неподвижной точке $\rho - F$-сжатия

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Аннотация. В работе исследуется вопрос об условиях существования и единственности неподвижной точки отображения полного метрического пространства. Вначале обсуждаются понятия F-сжатия и F^*-сжатия в теории неподвижных точек. Эти понятия, разработанные соответственно Вардовским и Пири совместно с Кумамом, послужили катализатором значительных исследований в различных метрических пространствах. Затем предлагаются обобщения этих понятий — $\rho - F$-сжатие и $\rho - F^*$-сжатие, демонстрируется их эффективность в обеспечении существования и единственности неподвижных точек. Этот новый подход обеспечивает большую гибкость за счет использования функции ρ, которая регулирует сжатие, расширяя возможности применения F- и F^*-сжатий. Завершает статью пример отображения, являющегося $\rho - F$-сжатием и $\rho - F^*$-сжатием и имеющего единственную неподвижную точку. При этом это отображение не удовлетворяет условиям Вардовского и условиям Пири и Кумама.

Ключевые слова: неподвижная точка, существование, единственность, F-сжатие, $\rho - F$-сжатие

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Introduction

The introduction of two distinct approaches to the concept of F-contraction and F^*-contraction, whereas F^* stands out as an adaptation of the F-contraction [1], developed respectively by Wardowski [2] and Piri with Kumam on [3] and authors on [4], has catalyzed significant research in the field of fixed point theories [5]. These approaches have found extensive applications in a multitude of metric spaces, such as b-metric, conic, partial, and fuzzy spaces, among others [6–10]. Essentially, this concept guarantees the existence of a fixed point $S : X \to X$, where (X, d) represents a complete metric space. The F and F^* have the possibility of assimilating these two contractions to well-established contractions such as Boyd–Wong [11] and Matkowski [12], for this, we must ensure certain conditions. Once S satisfies the following property, known as the F-contraction mapping:

$$\exists\, \tau > 0, \forall x, y \in X,\ Sx \neq Sy,\ F(d(Sx, Sy)) + \tau < F(d(x, y)),$$

where $F : [0, +\infty) \to \mathbb{R}$ is assumed to satisfy the following $F[\mathbb{R}]$ conditions: [1]

- (1_F): F is strictly increasing, i.e, $0 < t < s \Rightarrow F(t) < F(s)$,
- (2_F): $\lim_{t\to 0^+} F(t) = -\infty$,
- (3_F): There exists $k \in (0, 1)$, such that $\lim_{t\to 0^+} t^k F(t) = 0$,

or we use the $F^*[\mathbb{R}]$ conditions [12], which means that F verifies (1_F), (2_F) and

- (3_{F^*}): F is a continuous function in $(0, +\infty)$.

However, while this theory represents great mathematical interest, it has seen numerous different applications and a strong attraction for scientific research. We find that Awais et al [13] establish fixed-point results for F-contractions of Reich type for single-valued and multivalued applications in complete metric spaces. Sahil et al [14] introduced the notion of generalized F-contraction and established fixed point theorems for this type of functions in complete metric spaces. They also explore F-expansions and their applications. We find also Inayat et al [15] establish fixed-point results for generalized F-contractions in complete metric spaces. They generalize and unify several known results in the literature. But, Meir Keeler [16] addresses fixed-point results for a class of contractions in metric spaces. He demonstrates that F-contractions (and $F*$-contractions) fall under the category of Meir–Keeler contractions. Many results derived from F-contractions also apply to Meir–Keeler mappings.

Whereas, Zhukovskiy [17] extends the fixed point theory to f-quasimetric spaces, particularly generalizing the concept of F-contraction. Unlike the classic F-contraction proposed by Wardowski, where the parameter τ is constant, the article introduces a generalized version where τ is a variable function that depends on $F(d(x, y))$. As illustrated in [17, example 7], this new formulation allows τ to vary based on the distance between two points x and y, providing greater flexibility in analyzing contractions within nonsymmetric metric spaces. This generalization maintains key results, such as the existence of a unique fixed point and convergence of iterations towards that point, while broadening the scope to f-quasimetric spaces.

Our objective is to introduce a new class of contraction, $\rho-F$-contraction condition defined, for $\rho:\mathbb{R}\to\mathbb{R}$, as follows:

$$\forall x,y\in X \;\; d(x,y)>0 \;\Rightarrow\; F(d(Sx,Sy))\le \rho(F(d(x,y))).$$

This relation only indicates that it can be a generalization of F-contraction with $\rho(t)=t-\tau$, $t\in\mathbb{R}$. These assumptions are as follows:

- (1_R): ρ is increasing,
- (2_R): ρ is continuous,
- (3_R): For all $t\in\mathbb{R}$, $\rho(t)<t$.

In this work, we will demonstrate that the hypotheses $(1_R),(2_R),(3_R)$ are sufficient with the class $F[\mathbb{R}]$ and $F^*[\mathbb{R}]$ to ensure the existence of a fixed point for an operator S.

1. Preliminaries

Our goal is to demonstrate that if S is a $\rho-F$-contraction, where F is either of class $F[\mathbb{R}]$ or of class $F^*[\mathbb{R}]$ and ρ satisfies (1_R), (2_R) and (3_R), then S has a unique fixed point. But before beginning all this, we need the result presented by the following lemma.

Lemma 1.1. *Suppose that $\rho:\mathbb{R}\to\mathbb{R}$ is verifying $(1_R),(2_R)$ and (3_R), then,*

$$\forall t\in\mathbb{R} \qquad \lim_{n\to+\infty}\rho^n(t)=-\infty.$$

P r o o f. Suppose that there exists $t_0\in\mathbb{R}$, such that,

$$\lim_{n\to+\infty}\rho^n(t_0)=M.$$

We define the sequence $\{t_n\}_{n\ge 0}$ by t_0 chosen in $\mathbb{R}$ and $t_{n+1}=\rho(t_n)$, for all $n\ge 0$. This means that $\lim_{n\to+\infty}t_n=M$. Using (2_R) we get,

$$\lim_{n\to+\infty}t_{n+1}=\rho(\lim_{n\to+\infty}t_n) \;\Rightarrow\; M=\rho(M).$$

And this is a contradiction with (3_R). □

2. $\rho-F^*$-contraction

In this section, we show that the class of function $\rho[\mathbb{R}]$ is compatible with the $F^*[\mathbb{R}_+]$ to ensure the existence and the unicity for a mapping S through the new condition $\rho-F$-contraction.

Theorem 2.1. *Let (X,d) be a complete metric space. Let $S:X\to X$ and for ρ verifying (1_R), (2_R) and (3_R), $F\in F^*[\mathbb{R}]$,*

$$\forall x,y\in X \;\; d(x,y)>0 \;\Rightarrow\; F(d(Sx,Sy))\le \rho(F(d(x,y))).$$

Then, S has a unique fixed point $x^\infty\in X$ and

$$\forall x_0\in X \;\; \lim_{n\to+\infty}S^n x_0=x^\infty.$$

P r o o f. For $x_0 \in X$, we define the sequence $\{x_n\}_{n\geq 0}$ given by $x_{n+1} = Sx_n$ for all $n \geq 0$. We remark that if exists $n_0 \geq 0$, $x_{n_0} = Sx_{n_0}$ then x_{n_0} will be the fixed point x^∞. Now, suppose that for all $n \geq 0$, $d(x_n, Sx_n) > 0$. We obtain,

$$F(d(x_n, Sx_n)) = F(d(Sx_{n-1}, Sx_n)) \leq \rho(F(d(x_{n-1}, x_n))).$$

Applying (1_ρ) n-times we find

$$F(d(x_n, Sx_n)) \leq \rho(F(d(x_{n-1}, x_n))) \leq \rho^2(F(d(x_{n-2}, x_{n-1}))) \leq \ldots \leq \rho^n(F(d(x_0, x_1))).$$

Then, using Lemma 1.1,

$$\lim_{n\to+\infty} F(d(x_n, Sx_n)) = \lim_{n\to+\infty} \rho^n(F(d(x_0, x_1))) = -\infty,$$

and by (2_F) we conclude that

$$\lim_{n\to+\infty} d(x_n, Sx_n) = 0.$$

Now to obtain the fixed point we have to show that $\{x_n\}_{n\geq 0}$ is a Cauchy sequence.

We suppose the existence of some $\varepsilon > 0$ and two sequences of natural numbers $\{\varphi(n)\}_{n\geq 0}$ and $\{\psi(n)\}_{n\geq 0}$ such that:

$$\forall n \geq 0 \quad \varphi(n) > \psi(n) > n, \quad d(x_{\varphi(n)}, x_{\psi(n)}) \geq \varepsilon, \quad d(x_{\varphi(n)-1}, x_{\psi(n)}) < \varepsilon.$$

We obtain,

$$\begin{aligned} \varepsilon &\leq d(x_{\varphi(n)}, x_{\psi(n)}) \leq d(x_{\varphi(n)}, x_{\varphi(n)-1}) + d(x_{\varphi(n)-1}, x_{\psi(n)}) \\ &\leq d(x_{\varphi(n)}, x_{\varphi(n)-1}) + \varepsilon = d(x_{\varphi(n)-1}, Sx_{\varphi(n)-1}) + \varepsilon. \end{aligned}$$

But,

$$\lim_{n\to+\infty} d(x_{\varphi(n)-1}, Sx_{\varphi(n)-1}) = 0 \;\Rightarrow\; \lim_{n\to+\infty} d(x_{\varphi(n)}, x_{\psi(n)}) = \varepsilon.$$

On the other hand and using $\lim_{n\to+\infty} d(x_n, Sx_n) = 0$, we can conclude the existence of $n_0 \geq 0$,

$$\forall n \geq n_0 \quad d(x_{\varphi(n)}, Sx_{\varphi(n)}) < \frac{\varepsilon}{4}, \quad d(x_{\psi(n)}, Sx_{\psi(n)}) < \frac{\varepsilon}{4}.$$

Now, we show that

$$\forall n \geq n_0 \quad d(x_{\varphi(n)+1}, x_{\psi(n)+1}) = d(Sx_{\varphi(n)}, Sx_{\psi(n)}) > 0. \tag{2.1}$$

In fact, suppose $\exists m \geq n_0$, such that

$$d(x_{\varphi(m)+1}, x_{\psi(m)+1}) = 0.$$

Then,

$$\begin{aligned} \varepsilon &\leq d(x_{\varphi(m)}, x_{\psi(m)}) \leq d(x_{\varphi(m)}, Sx_{\varphi(m)}) + d(x_{\varphi(m)+1}, x_{\psi(m)+1}) + d(Sx_{\psi(m)}, x_{\psi(m)}) \\ &\leq \frac{\varepsilon}{4} + 0 + \frac{\varepsilon}{4} = \frac{\varepsilon}{2}. \end{aligned}$$

Which is a contradiction. From (2.1) and using $\rho - F$-contraction hypothesis, we obtain

$$F(d(Sx_{\varphi(n)}, Sx_{\psi(n)})) \leq \rho(F(d(x_{\varphi(n)}, x_{\psi(n)}))).$$

Then

$$F(\varepsilon) \leq \rho(F(\varepsilon)) < F(\varepsilon).$$

The last contradiction allows us to confirm that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence converging to x^∞. Then

$$d(Sx^\infty, x^\infty) = \lim_{n\to+\infty} d(Sx_n, x_n) = 0 \;\Rightarrow\; x^\infty = Sx^\infty.$$

Suppose now that there exists another fixed point for S denoted y^∞, i.e. $y^\infty = Sy^\infty$, and $x^\infty \neq y^\infty \;\Rightarrow\; d(x^\infty, y^\infty) > 0$. Now,

$$F(d(x^\infty, y^\infty)) = F(d(Sx^\infty, Sy^\infty)) \leq \rho F(d(x^\infty, y^\infty)) < F(d(x^\infty, y^\infty)),$$

which is a contradiction, then S has a unique fixed point x^∞. □

3. $\rho - F$-contraction

Our objective now is to show that the family of conditions (ρ_1), (ρ_2) and (ρ_3) ensure, for a $\rho - F$-contraction mapping S, a unique fixed point, when F is in the class $F[\mathbb{R}_+]$. To achieve our goal, we add a condition on ρ :

$$\forall \beta > 1 \; \forall t \in \mathbb{R} \sum_{n\geq P(t)} |\rho^n(t)|^{-\beta} < \infty, \tag{3.1}$$

where $P(t)$ is the smaller natural such that, $\rho^{P(t)}(t) < 0$.

Theorem 3.1. *Let (X, d) be a complete metric space and $S : X \to X$ be a $\rho - F$-contraction. Then S has a unique fixed point x^∞, i. e.*

$$x^\infty = Sx^\infty \;\; \textit{and} \;\; \forall x_0 \in X \lim_{n\to+\infty} S^n x_0 = x^\infty.$$

P r o o f. For $x_0 \in X$, $n \in \mathbb{N}$, we introduce the positif real sequence $\{a_n\}_{n\geq 0}$ by

$$a_n = d(x_n, Sx_n) = d(x_n, x_{n+1}),$$

where $\{x_n\}_{n\geq 0} \subset X$ is given by $x_0 \in X$ and $x_{n+1} = Sx_n \;\; \forall n \geq 0$. If there exists $n_0 \geq 0$ such $a_{n_0} = 0$, then $x_{n_0} = Sx_{n_0}$ is the fixed point.

Now, suppose that $a_n > 0$, for all $n \geq 0$. Using the $\rho - F$-contraction hypothesis, we obtain

$$F(a_n) \leq \rho(F(a_{n-1})) \leq \rho^2(F(a_{n-2})) \leq \ldots \leq \rho^n(F(a_0)).$$

Then,

$$\lim_{n\to+\infty} F(a_n) = \lim_{n\to+\infty} \rho^n(F(a_0)) = -\infty,$$

and

$$\lim_{n\to+\infty} a_n = 0.$$

Let

$$n \geq p(F(a_0)),$$

then

$$a_n^k F(a_n) \leq a_n^k \rho^n(F(a_0)) \leq 0 \;\Rightarrow\; \lim_{n\to+\infty} a_n^k \rho^n(F(a_0)) = 0.$$

We take $n \geq N \geq P(F(a_0))$, N bigger enough such that,

$$|a_n^k \rho(F(a_0))| \leq 1 \;\Rightarrow\; a_n \leq |\rho^n(F(a_0))|^{-\frac{1}{k}},$$

then, for all $m > n > N$,

$$d(x_m, x_n) \leq d(x_m, x_{m-1}) + d(x_{m-1}, x_{m-2}) + \ldots + d(x_{n+1}, x_n) \leq \sum_{n \geq N} |\rho^n(F(a_0))|^{-\frac{1}{k}} \to 0.$$

Then, $\{x_n\}_{n\geq 0}$ is a Cauchy sequence and it is converging to x^∞ in X. It is clear that x^∞ is a fixed point of S. Suppose now that $y^\infty \in X$ is also a fixed point of S. Then,

$$F(d(x^\infty, y^\infty)) = F(d(Tx^\infty, Ty^\infty)) \leq \rho(F(d(x^\infty, y^\infty))) < F(d(x^\infty, y^\infty)),$$

which is a contraction. Then, S has a unique fixed point. □

4. Illustration

Let $X = \{x_n \mid n = 1, 2, \ldots, \infty\}$ and define the metric $d : X \to X$ as follows: for $m > n$ $d(x_n, x_m) = d(x_m, x_n) = \frac{1}{n}$, and $d(x_n, x_n) = 0$.

Let $S : X \to X$ be defined by $S(x_n) = x_{n+1}$. Then we have:

$$d(S(x_n), S(x_m)) = \frac{1}{n+1},$$

and

$$\frac{d(S(x_n), S(x_m))}{d(x_n, x_m)} = \frac{n}{n+1} \to 1, \quad n \to \infty.$$

Therefore, S is not an ordinary contraction. Let's define the function $F(d) = \ln d$, which satisfies the conditions $F[\mathbb{R}]$ and $F^*[\mathbb{R}]$.

Now, let's compute τ :

$$\tau = -F(d(S(x_n), S(x_m))) + F(d(x_n, x_m)) = -\ln \frac{1}{n+1} + \ln \frac{1}{n} = \ln \frac{n+1}{n} \to 0, \quad n \to \infty.$$

This implies that S does not satisfy the conditions of the Wardowski and Piri–Kumam theorems (where $\tau > 0$). Let us show that S satisfies the conditions of the theorems proven in this article.

Now, let the function $\rho(t) = \ln\left(\frac{e^t}{e^t+1}\right) = t - \ln(e^t + 1)$. This function is increasing as:

$$\rho'(t) = 1 - \frac{e^t}{e^t + 1} = \frac{1}{e^t + 1} > 0.$$

Furthermore, $\rho(t) < t$ for $t > 0$, and ρ is continuous. Thus, ρ verified all conditions (ρ_1), (ρ_2) and (ρ_3).

Finally, we have:

$$\rho(F(d(x_n, x_m))) = \rho\Big(F\Big(\frac{1}{n}\Big)\Big) = \rho\Big(\ln \frac{1}{n}\Big) = \ln\Big(\frac{e^{\ln \frac{1}{n}}}{e^{\ln \frac{1}{n}} + 1}\Big) = \ln \frac{1}{n+1} = F(d(Sx_n, Sx_m)).$$

We have introduced the concept of $\rho - F$-contraction, which extends the classic F-contraction. This new approach is crucial because it generalizes fixed-point results in metric spaces by incorporating a function ρ that adjusts the contracting behavior of the F-function. This generalization is important as it not only covers traditional F-contractions but also broader versions like those of Wardowski and Piri–Kumam, offering greater flexibility in constructing unique fixed points and analyzing contractive behaviors in various contexts.

As a future direction, it would be interesting to apply this notion of $\rho - F$-contraction to f-quasimetric spaces [17], where distances are not necessarily symmetric. This would open up the exploration of new classes of contractive mappings and further validate the effectiveness of this generalized approach in more complex spaces, where distances may be asymmetric or only partially defined.

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