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Probabilistic model of surface layer removal when grinding brittle non-metallic materials

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ABSTRACT

Introduction. The final quality of products is formed during finishing operations, which include the grinding process. It is known that when grinding brittle materials, the cost of grinding work increases significantly. It is possible to reduce the scatter of product quality indicators when grinding brittle materials, as well as to increase the reliability and efficiency of the operation, by choosing the optimal parameters of the technological system based on dynamic models of the process. However, to describe the regularities of the removal of particles of a brittle non-metallic material and the wear of the surface of the grinding wheel in the contact zone, the known models do not allow taking into account the peculiarities of the process in which micro-cutting and brittle chipping of the material are combined. **Purpose of the work:** to create a new probabilistic model for removing the surface layer when grinding brittle non-metallic materials. The task is to study the laws governing the removal of particles of brittle non-metallic material in the contact zone. In this work, the removal of material in the contact zone as a result of microcutting and brittle chipping is considered as a random event. **The research methods** are mathematical and physical simulation using the basic provisions of the theory of probability, the laws of distribution of random variables, as well as the theory of cutting and the theory of a deformable solid. **Results and discussion.** The developed mathematical models make it possible to trace the effect on material removal of the overlap of single cuts on each other when grinding holes in ceramic materials. The proposed dependences show the regularity of stock removal within the arc of contact of the grinding wheel with the workpiece. The considered features of the change in the probability of material removal upon contact of the treated surface with an abrasive tool and the proposed analytical dependences are valid for a wide range of grinding modes, wheel characteristics and a number of other technological factors. The obtained expressions make it possible to find the amount of material removal also for schemes of end, flat and circular external grinding, for which it is necessary to know the amount of removal increment due to brittle fracture during the development of microcracks in the surface layer. One of the ways to determine the magnitude of this increment is to simulate the crack formation process using a computer. The presented results confirm the prospects of the developed approach to simulate the processes of mechanical processing of brittle non-metallic materials.

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Introduction

Structural non-metallic materials such as ceramics, glass, quartz, ferrites, sitalls are increasingly used in industry due to its high hardness, strength and wear resistance. However, these materials are also highly fragile, which makes it much more difficult to process. The quality parameters of products, which determine its functional suitability and performance characteristics, are finally formed at finishing operations, which include the grinding process. It is known that when grinding metals, the cost of grinding work takes on

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average 15 ... 25% of the total cost of manufacturing products [1], when processing brittle materials, this figure increases significantly.

Grinding processes have a complex stochastic nature [2], which leads to a scatter of product quality indicators, a decrease in the reliability, productivity and economy of the technological process. It is possible to reduce the scatter of product quality indicators when grinding brittle materials, as well as to increase the reliability and efficiency of the operation, by choosing the optimal parameters of the technological system based on dynamic models of the process.

A large number of studies are devoted to the development of dynamic models for various processes of abrasive processing [3-15]. However, to describe the regularities of the removal of particles of a brittle non-metallic material and the wear of the surface of the grinding wheel in the contact zone, the known models do not allow taking into account the peculiarities of the process in which micro-cutting-chipping and brittle volume fracture of the material are combined. In this regard, the aim of the work is to create a new probabilistic model for removing the surface layer when grinding brittle non-metallic materials. The task is to study the laws governing the removal of particles of brittle non-metallic material in the contact zone.

Simulation of the process

To obtain dependencies that allow calculating material removal when grinding holes in workpieces made of brittle non-metallic materials, consider the presented scheme (Fig. 1).

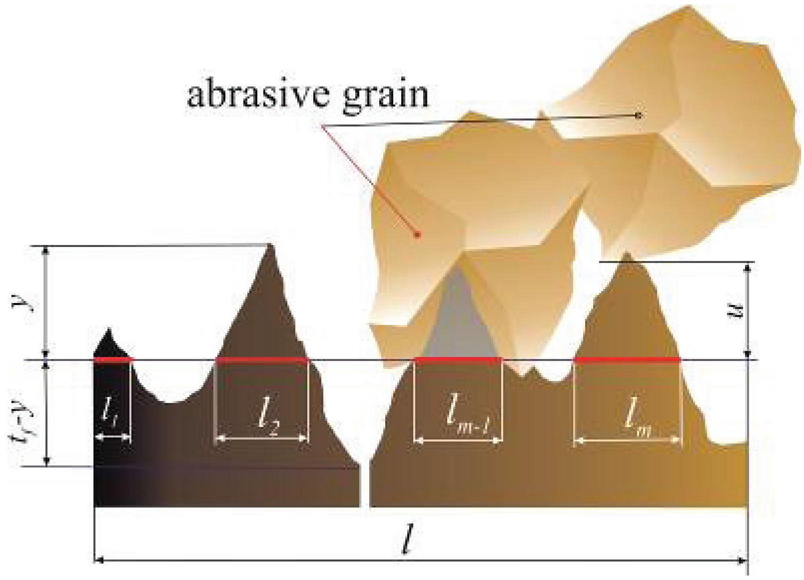

Fig. 1. Scheme for calculating the probability of removing the surface layer when grinding the material

In the period under consideration, the most protruding peaks of abrasive grains pass through the level of the surface of the workpiece, which, in contact with the treated surface, leave traces on it in the form of scratches. Moreover, the probability of its overlap can be full or partial. In most cases, incomplete contact is predominantly observed. Part of the grains of the abrasive tool can fall into the trail of the preceding grains, leaving no scratches.

An analysis of the study of the mechanisms of material removal by a single grain shows that when passing through the contact zone, the grain can cut off the material when it hits the protrusion of the surface roughness, or it can not cut the specified material when passing through the cavity of the rough surface. Based on the analysis of the contact of the top of the abrasive grain with the material, a theoretical-probabilistic model is proposed in [16] that allows calculating the amount of material removal when grinding plastic materials. The probability is determined by the ratio of the unremoved part of the metal to the total length of the section under consideration (Fig. 1):

$$P_k = \lim_{l\to\infty} \frac{\sum_{i=1}^{n} l_i}{l} = 1 - P(M) = \exp(-a_0 - a), \quad (1)$$

where: a_0 – an indicator characterizing the initial state of the surface of the workpiece in a given section before the start of the grinding process; a – an indicator characterizing the change in the area of the depressions formed by the sum of the profiles of abrasive grains passing through the considered section of the workpiece; $P(M)$ – probability of material removal.

To describe the regularities of material removal and tool surface wear in the contact zone, the concepts of the probability of removal $P(M)$ and the probability of non-removal $P(\overline{M})$ of the material are proposed in [17]. The first indicator $P(M)$ is determined by the probability of an event in which material at a point on the treated surface is removed. The second indicator $P(\overline{M})$ is the probability of an event in which material is not removed from the treated surface. The sum of the probabilities, as the probabilities of opposite events, is equal to unity, and its values depend on the position of the point in the contact zone. For the processes of processing workpieces with abrasive tools, the probability of material removal is calculated from the dependence:

$$P(M) = 1 - e^{-(a_0 + a_1 + \ldots + a_k + \ldots + a_j)}, \quad (2)$$

where $a_1, \ldots, a_j$ – are indicators characterizing the change in the areas of the depressions formed by the sum of the profiles of abrasive grains passing through the considered section of the workpiece after the corresponding contacts of the grains with the surface of the workpiece.

In general, when finishing and fine grinding of holes in workpieces made of brittle non-metallic materials (glass, ceramics, quartz, ferrites, sitalls, etc.), as well as in workpieces with ceramic coatings, material removal is carried out due to a combination of micro-cutting-chipping and brittle volume destruction of the material.

To obtain a mathematical model that allows calculating removal when grinding brittle non-metallic materials, consider the process of contacting the tool with the workpiece at the level y.

As a result, of the impact of cutting and piercing grains on the surface of the workpiece, material is removed in the contact zone by micro-cutting and brittle chipping, which can be considered as a random event. It is characterized by the joint probability of material removal from the workpiece by the micro-cutting or shearing process. Thus, the probability of removal when grinding brittle non-metallic materials, is calculated by the formula:

$$P(M) = P_1(\overline{M}) \cdot P_2(\overline{M}), \quad (3)$$

where $P_1(\overline{M})$ – is the probability of an event in which the processed material is not removed due to the micro-cutting process; $P_2(\overline{M})$ – the probability of an event in which the processed material is not removed due to the brittle chipping process.

Similarly to equation (1), dependence (3) can be described by the following expression:

$$P(M) = 1 - \exp(-a_0 - a_1(y, \tau) - a_2(y, \tau)), \quad (4)$$

where a_0 – an indicator characterizing the initial state of the surface of the workpiece in a given section before the start of the grinding process; $a_1(y, \tau)$ – an indicator characterizing the change in the area of the depressions formed due to the mechanical cutting process; $a_2(y, \tau)$ – an indicator characterizing the change in the area of depressions formed due to the process of brittle chipping; y – distance from the outer surface of the workpiece to the current level; τ – the moment in time of an ongoing event.

For each revolution (pass), the change in the increment of the indicator Δa_1 is determined by the expression [16]:

$$\Delta a_1(y,\tau) = k_c \cdot b_z \cdot \Delta\lambda, \tag{5}$$

where k_c – is the chip formation coefficient; $b_z(y,\tau)$ – is grain width at the level y; $\Delta\lambda$ – is the number of abrasive grains that have passed through the section under consideration.

When grinding brittle materials, the chip formation coefficient k_c is 1, since there are no plastic deformation processes. To calculate the indicator Δa_1, characterizing the change in the area of the depressions formed due to the mechanical cutting process, grains that cut off the material are taken into account (piercing grains are not considered in this case). Based on this, taking into account equation (5), the indicator Δa_1 can be calculated as follows:

$$\Delta a_1(y,\tau) = b_z \cdot \Delta\lambda \cdot (1 - P_{ck}), \tag{6}$$

where P_{ck} – is the probability of brittle chipping of the workpiece material.

Through a single section of the surface, with a thickness of Δu (Fig. 2), the tops of the abrasive grains $\Delta\lambda$ will pass over time $\Delta\tau$. The number of tops of abrasive grains can be calculated from the density of its distribution in the working layer of the tool $f(u)$ along the coordinate u:

$$\Delta\lambda = n_z \cdot f(u) \cdot \Delta u \cdot (V_k \pm V_u) \cdot \Delta\tau, \tag{7}$$

where n_z – is the number of grains per unit area of the working layer of the tool; V_k – the peripheral speed of the tool (circle); V_u – the peripheral speed of the workpiece.

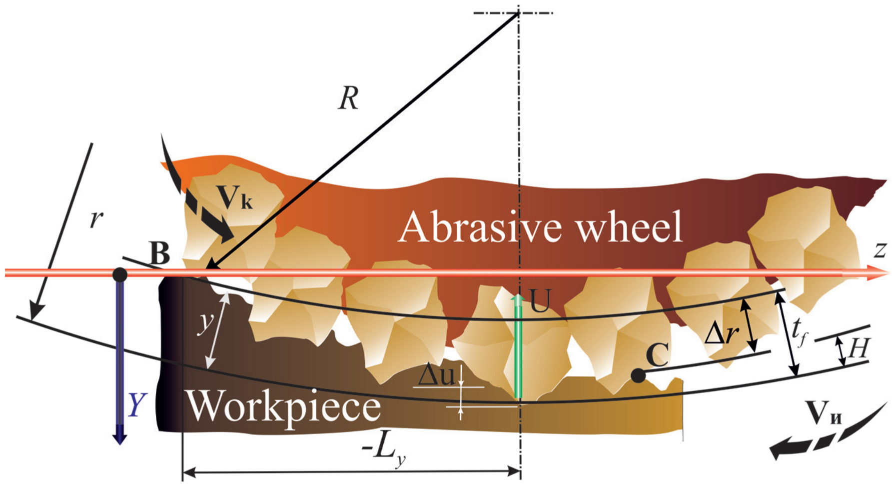

Fig. 2. Scheme for calculating the number of tops of abrasive grains, passing through a unit surface area by the thickness of the tool per unit of time

The distribution of cutting edges over the depth of the working surface of the tool was studied in [2, 15, 18]. In the analytical description of the distribution curves J. Cassen assumes that the number of cutting edges on the surface of the circle is proportional to the square of the distance inside the circle [19].

The probability density curve of the distribution of cutting edges is modeled by its straight-line dependence $f(u) = C_f \cdot u$. According to the author, the simulation of the distribution curve by a straight-line dependence is valid for the section of the circle directly lying near the surface. To describe the distribution density of the tops of abrasive grains, O.Coyle suggested using a dependence of the form [17]:

$$f(u) = C_h \cdot u^{\chi-1}, \tag{8}$$

where C_h – is the coefficient of proportionality of the distribution curve:

$$C_h = \frac{\chi}{H_u^{\chi}},$$

where H_u – is the thickness of the layer of the working surface of the tool in contact with the workpiece.

Taking into account the above, dependence (8) can be represented as:

$$f(u) = \frac{\chi}{H_u^{\chi}} \cdot u^{\chi-1}, \tag{9}$$

where χ – is the parameter of the distribution density function.

Comparison of the values of the probability density of the distribution for different models (Fig. 3) indicates that the most significant difference from the dynamic distribution has a straight-line relationship. The best approximation is provided by the power-law dependence of the modified Γ-distribution function.

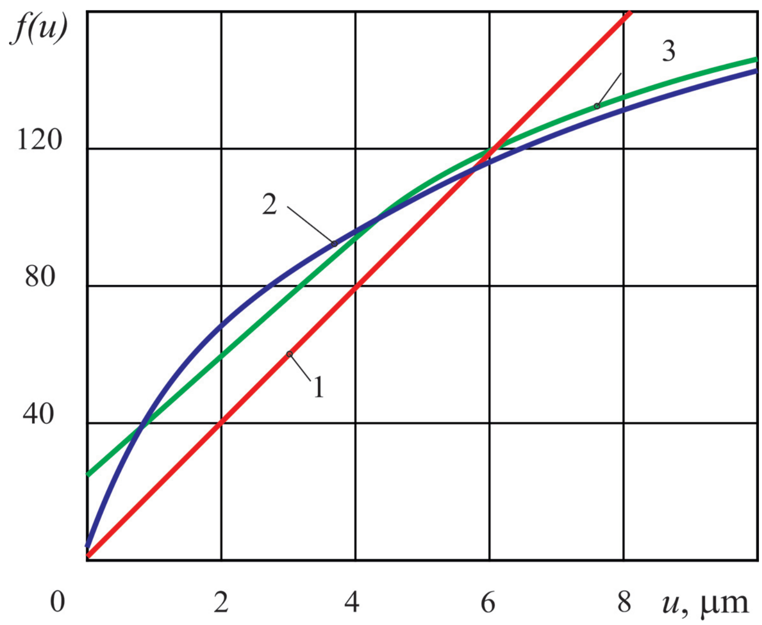

Fig. 3. Simulation the probability density of the distribution of the tops of grains when approximating their profile:

1 – straight-line dependence; 2 – a parabola; 3 – modified function Γ-distributions

Power-law dependences are currently widely used not only for the mathematical description of the distribution of grain tops on the working surface of grinding tools.

To characterize the shaping process, it is also of considerable interest to calculate the number of abrasive grains passing through an elementary surface area.

The increment in the number of grains in general form is determined by equation (7), which, after substitution of values $f(u)$ and transition from a discrete model to a continuous one, takes the form:

$$\lambda(t) = \frac{(V_k \pm V_u) n_z \chi}{H_u^{\chi}} \int_0^t \left(t_f - y - \frac{z^2}{D_e} \right)^{\chi} d\tau. \tag{10}$$

After changing the variable τ to $\frac{z}{V_u}$ and integrating over, a dependence is obtained for χ calculating the current value of the number of cutting edges, passing through the section at $\chi = 1.5$:

$$\lambda(t) = \frac{(V_k \pm V_u) n_z \chi}{4 V_u H_u^{1.5} (\frac{dD}{d-D})^{1.5}} \times \left(z\left(L_y^2 - z^2\right)^{1.5} + \left(\frac{3L_y^2}{2}\left(z\left(L_y^2 - z^2\right)^{1.5} + \right.\right.\right.$$

$$\left.\left.\left. + L_y^2 \left(\arcsin\left(\frac{z}{L_y}\right) + \frac{\pi}{2}\right)\right)\right)\right) \tag{11}$$

The number of cutting edges that pass during the contact of the section with the circle is determined from equation (11) at the upper limit of integration $z = L_y$:

$$\lambda = \frac{\sqrt{\pi D_e} \cdot \Gamma(\chi)\chi(V_k \pm V_u) n_z}{\Gamma(\chi + 3/2) V_u H_u^{\chi}} (t_f - y)^{\chi + 0.5}. \tag{12}$$

The width of the grain profiles in the working layer of the tool at the level y from the surface of the workpiece will be equal to:

$$b_z = C_b h^m = C_b (t_f - y - u)^m, \tag{13}$$

where C_b, m – is the proportionality coefficient and the exponent, respectively, in the equation when the grain shape is approximated by a paraboloid of revolution; t_f – is an actual depth of cut; u – is the position of the grain in the abrasive tool relative to its conditional outer surface (Fig. 4).

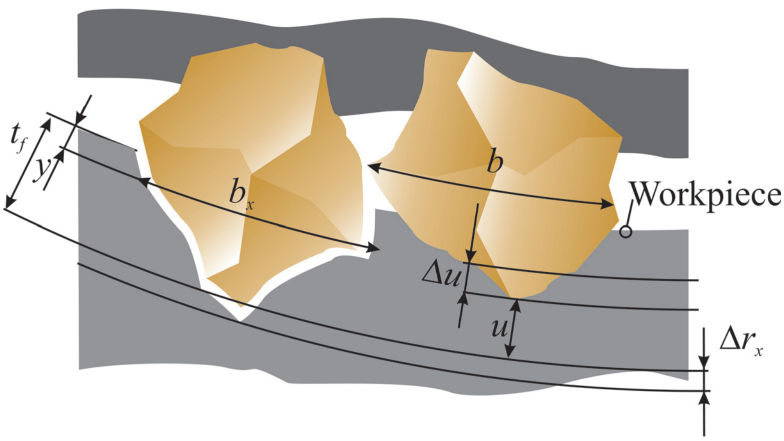

Fig. 4. Scheme of interaction of abrasive grains with a ceramic workpiece

After substituting expressions (9) and (13) into (6), the dependence for calculating the indicator $\Delta a_1(y, \tau)$ is obtained:

$$\Delta a_1(y, \tau) = k_c \cdot n_z \cdot b_z \cdot f(u) \cdot \Delta u \cdot (V_k \pm V_u) \cdot (1 - P_{ck}) \cdot \Delta\tau. \tag{14}$$

We replace the variable with τ on $\frac{z}{V_u}$ and after substituting it into expression (14) we get:

$$a_1(y, z) = \int_{-L_y}^{z} \int_{0}^{t(z)-y} k_c \cdot n_z \cdot b_z \cdot f(u) \cdot \frac{(V_k \pm V_u)}{V_u} \cdot (1 - P_{ck}) du dz, \tag{15}$$

where L_y – is the length of the contact zone from the conditional outer surface of the tool to the main plane (Fig. 2), which can be calculated from the dependence:

$$L_y = \sqrt{(t_f - y) \cdot D_e}\,. \tag{16}$$

To calculate the probability of an event characterizing the chipping process of the workpiece material P_{ck} during grinding, the following relationship was used [20]:

$$P_{ck} = P_o \cdot \left[1 - (\frac{u}{t_f})^{\beta}\right], \tag{17}$$

where P_0 – is the probabilistic characteristic of chipping of a brittle non-metallic material chipping; β – is the exponent in the probability equation. The indicated parameters of dependence (17) can be calculated by the method, proposed in [21].

When substituting the obtained expressions b_z and $f(u)$ into equation (15) and from equations (13) and (9), it takes the form:

$$a_1(y,z) = \frac{k_c C_b (V_k \pm V_u) n_z \chi}{V_u H_u^{\chi}} \int_{-L_y}^{z} \int_{0}^{t(z)-y} \left(t_f - y - u - \frac{z^2}{D_e}\right)^m u^{\chi-1} du dz -$$
$$-\frac{k_c C_b (V_k \pm V_u) n_z \chi P_0}{V_u H_u^{\chi}} \int_{-L_y}^{z} \int_{0}^{t(z)-y} \left(t_f - y - u - \frac{z^2}{D_e}\right)^m u^{\chi-1} du dz +$$
$$+\frac{k_c C_b (V_k \pm V_u) n_z \chi P_0}{V_u H_u^{\chi}} \int_{-L_y}^{z} \int_{0}^{t(z)-y} \left(t_f - y - u - \frac{z^2}{D_e}\right)^m u^{\chi-1} \left(\frac{u}{t_f}\right)^{\beta} du dz\,. \tag{18}$$

Results and discussion

The previously adopted models of the grain tops and its depth distribution densities make it possible to proceed to the establishment of functional relationships between the probability of non-removal of the material and technological factors. After integrating the resulting equation over u we get:

$$a_1(y,z) = \frac{k_c C_b (V_k \pm V_u) n_z \chi \Gamma(\chi)\Gamma(m+1)(1-P_0)}{\Gamma(m+\chi+1) V_u H_u^{\chi}} \int_{-L}^{z} \left(t_f - y - \frac{z^2}{D_e}\right)^{m+\chi} dz +$$
$$+\frac{k_c C_b (V_k \pm V_u) n_z \chi \Gamma(\chi+\beta)\Gamma(m+1)}{\Gamma(m+\chi+\beta+1) V_u H_u^{\chi} t_f^{\beta}} \int_{-L}^{z} \left(t_f - y - \frac{z^2}{D_e}\right)^{m+\chi+\beta} dz\,, \tag{19}$$

where Γ (...) – are the corresponding gamma functions.

Integration of equation (19) is possible only for particular values of the coefficients. For $\chi = 1.5$, $m = 0.5$, $\beta = 2$ and $C_b = 2\sqrt{2 \cdot \rho_z}$ we get:

$$a_1(y,z) = \frac{3k_c C_b (V_k \pm V_u) n_z \Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}\right)(1-P_0)(t_f - y)^2}{2\Gamma(3) V_u H_u^{\frac{3}{2}}} \times \left(z - \frac{2z^3}{3\sqrt{L_y}} + \frac{z^5}{5L_y} + \frac{8}{15}L_y\right) +$$
$$+\frac{3k_c C_b (V_k \pm V_u) n_z \Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{3}{2}\right)(t_f - y)^4}{2\Gamma(5) V_u H_u^{\frac{3}{2}} t_f^2} \left(z + \frac{z^9}{9L_y^2} - \frac{4z^7}{7L_y^{\frac{3}{2}}} + \frac{6z^5}{5L_y} - \frac{4z^3}{3\sqrt{L_y}} + \frac{8}{20}L_y\right). \tag{20}$$

After substituting the values of the gamma functions, we get:

$$a_1(y,z) = \frac{3\pi k_c \sqrt{2\rho_z}(V_k \pm V_u) n_z (1-P_0)(t_f - y)^2}{8V_u H_u^{\frac{3}{2}}} \left(z - \frac{2z^3}{3\sqrt{L_y}} + \frac{z^5}{5L_y} + \frac{8}{15} L_y \right) +$$

$$+ \frac{3\pi k_c \sqrt{2\rho_z}(V_k \pm V_u) n_z (t_f - y)^4}{16 V_u H_u^{3/2} t_f^2} \times \left(z + \frac{z^9}{9L_y^2} - \frac{4z^7}{7L_y^{3/2}} + \frac{6z^5}{5L_y} - \frac{4z^3}{3\sqrt{L_y}} + \frac{8}{20} L_y \right). \quad (21)$$

The calculation of the indicator $a_2(y,z)$, characterizing the change in the area of the depressions formed due to the brittle cleavage process in any area of the contact zone with the known initial state of the surface is calculated similarly to the indicator $a_1(y,z)$.

To calculate the indicator $a_2(y,z)$, it is necessary to take into account that the course of the brittle shearing process is accompanied by an increase in the width of the single risk b_z to the value b_x (Fig. 4). For approximation b_x, a power-law dependence was used:

$$b_x = C_{bx} \left(t_f - y - u + \Delta r_x - \frac{z^2}{D_e} \right)^{m_x}, \quad (22)$$

where Δr_x – is the increment in material removal in the process of brittle chipping of brittle non-metallic material; m_x – is the exponent in the equation, that simulates the shearing grain profile as a paraboloid of revolution. The depth distribution density of shear grains can be calculated using the formula:

$$f(u) = \frac{\chi_x}{H_u^{\chi_x}} u^{\chi_x - 1}, \quad (23)$$

where χ_x – is the parameter of the distribution density function of shear grains.

The dependence for calculating the indicator $a_2(y,z)$, included in the expression for calculating the probability of material removal due to volume brittle fracture, similar to the solution given above (18), is written as:

$$a_2(y,z) = \frac{k_c C_b (V_k \pm V_u) n_z \chi}{V_u H_u^{\chi}} \int_{-L_y}^{z} \int_{0}^{t(z)-y} \left(t_f - y - u - \frac{z^2}{D_e} \right)^m u^{\chi-1} du dz -$$

$$- \frac{k_c C_b (V_k \pm V_u) n_z \chi P_0}{V_u H_u^{\chi}} \int_{-L_y}^{z} \int_{0}^{t(z)-y} \left(t_f - y - u - \frac{z^2}{D_e} \right)^m u^{\chi-1} du dz +$$

$$- \frac{k_c C_b (V_k \pm V_u) n_z \chi P_0}{V_u H_u^{\chi}} \int_{-L_y}^{z} \int_{0}^{t(z)-y+\Delta r_x} \left(t_f - y - u - \frac{z^2}{D_e} + \Delta r_x \right)^{m_x} u^{\chi-1} \left(\frac{u}{t_f} \right)^{\beta} du dz. \quad (24)$$

After integrating expression (24) over u we get:

$$a_2(y,z) = \frac{k_c C_b (V_k \pm V_u) n_z \chi \Gamma(\chi) \Gamma(m+1)(1-P_0)}{\Gamma(m+\chi+1) V_u H_u^{\chi}} \int_{-L}^{z} \left(t_f - y - \frac{z^2}{D_e} \right)^{m+\chi} dz +$$

$$+ \frac{k_c C_b (V_k \pm V_u) n_z \chi \Gamma(\chi+\beta) \Gamma(m_x+1)}{\Gamma(m_x+\chi+\beta+1) V_u H_u^{\chi} t_f^{\beta}} \int_{-L}^{z} \left(t_f - y - \frac{z^2}{D_e} + \Delta r_x \right)^{m_x+\chi+\beta} dz. \quad (25)$$

After substituting the values of gamma functions at particular values $\chi = 1.3$, $m_x = 0.7$ and $\beta = 2$ into expression (25), we get:

$$a_2(y,z) = \frac{6k_c\sqrt{2\rho_z}(V_k \pm V_u)n_z\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}\right)(1-P_0)}{2\Gamma(3)V_u H_u^{\frac{3}{2}}} \int_{-L}^{z}\left(t_f - y - \frac{z^2}{D_e}\right)^2 dz +$$

$$+\frac{13k_c\sqrt{2\rho_z}(V_k \pm V_u)n_z\Gamma(3.3)\Gamma(1.7)}{5\Gamma(5)V_u H_u^{1.3}t_f^2} \int_{-L}^{z}\left(t_f - y - \frac{z^2}{D_e} + \Delta r_x\right)^4 dz . \quad (26)$$

After substituting the values of the gamma functions, we finally get:

$$a_2(y,z) = \frac{3\pi k_c\sqrt{2\cdot\rho_z}(V_k \pm V_u)n_z(1-P_0)(t_f - y)^2}{8V_u H_u^{\frac{3}{2}}} \times \left(z - \frac{2z^3}{3\sqrt{L_y}} + \frac{z^5}{5L_y} + \frac{8}{15}L_y\right) +$$

$$+\frac{0.05k_c\sqrt{2\rho_z}(V_k \pm V_u)n_z(t_f - y + \Delta r_x)^4}{V_u H_u^{1.3}(t_f + \Delta r_x)^2}$$

$$\times\left(z + \frac{z^9}{9L_y^2} - \frac{4z^7}{7L_y^{\frac{3}{2}}} + \frac{6z^5}{5L_y} - \frac{4z^3}{3\sqrt{L_y}} + \frac{8}{20}L_y\right). \quad (27)$$

Conclusions

The developed mathematical models make it possible to trace the effect on material removal of the overlap of single sections on each other when grinding holes in ceramic materials. The proposed dependences show the regularity of stock removal within the arc of contact of the grinding wheel with the workpiece. The considered features of the change in the probability of material removal upon contact of the treated surface with an abrasive tool and the proposed analytical dependences are valid for a wide range of grinding modes, wheel characteristics and a number of other technological factors [20, 22]. The obtained expressions make it possible to find the amount of material removal also for schemes of end, flat and circular external grinding, for which it is necessary to know the amount of removal increment due to brittle fracture during the development of micro-cracks in the surface layer. One of the ways to determine the magnitude of this increment is to simulate the crack formation process using a computer.

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Conflicts of Interest

The authors declare no conflict of interest.