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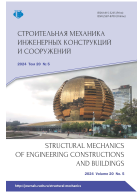

АНАЛИТИЧЕСКИЕ И ЧИСЛЕННЫЕ МЕТОДЫ РАСЧЕТА КОНСТРУКЦИЙ
ANALYTICAL AND NUMERICAL METHODS OF ANALYSIS OF STRUCTURES

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Strength Model for Concrete in Near-Reinforcement Region

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Abstract. The relevant problem of concrete strength in the near-reinforcement zone is solved as a problem of volumetric stress-strain state with the "closure" of output integral parameters of this zone on the framework of the whole reinforced concrete element, synthesizing hypotheses and dependencies of various disciplines of solid mechanics, including fracture mechanics. The model of reinforced concrete element takes into account Vl.I. Kolchunov's effect of reinforced concrete, which describes the mechanism of formation and development of transverse and longitudinal cracks. In this respect, generalized hypotheses of linear and shear strains for warping and gradients of relative mutual displacements of reinforcement and concrete are adopted. New functionals of reinforced concrete are constructed, which are consistent with the physical interpretations of the strength of cross-sections of bar elements in near-reinforcement zones. Constitutive equations for the concrete matrix, which models zones between transverse cracks, are written. The displacement components for the near-reinforcement zone in relation to the crack opening width at the "concrete-reinforcement" contact interface in transverse, longitudinal and radial cracks, respectively, are found. The use of the adopted assumptions and a multi-level calculation approach for the near-reinforcement region brings the model significantly closer to a real evaluation of the physical phenomena.

Keywords: volumetric stress state, near-reinforcement zone, displacement, cylindrical coordinates, effect of reinforced concrete, linear and shear strains, generalized hypothesis

Conflicts of interest. The authors declare that there is no conflict of interest.

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Расчетная модель сопротивления железобетона в околоарматурной области

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Аннотация. Решена актуальная задача сопротивления околоарматурной зоны бетона как задача объемного напряженно-деформированного состояния с «замыканием» выходных интегральных параметров этой зоны на стержневую схему всего железобетонного элемента, синтезирующую в себе гипотезы и зависимости механики железобетона и механики разрушения. В расчетной модели железобетонного элемента учтен эффект железобетона проф. Вл.И. Колчунова описывающий механизм образования и развития поперечных и продольных трещин. При этом приняты обобщенные гипотезы линейных и угловых деформаций для депланаций и градиентов относительных взаимных смещений арматуры и бетона. Построены новые функционалы железобетона, которые согласуются с физическими представлениями о сопротивлении поперечных сечений стержневых элементов в околоарматурных зонах. Записаны физические уравнения для бетонной матрицы, моделирующей зоны между поперечными трещинами. Найдены составляющие перемещений для околоарматурной области применительно к ширине раскрытия трещин на границе контакта «бетон–арматура» в поперечных, продольных и радиальных трещинах соответственно. Использование принятых предпосылок и многоуровневой расчетной схемы для околоарматурной области заметно приближает расчетную модель к реальной оценке физических явлений.

Ключевые слова: объемное напряженное состояние, околоарматурная зона, перемещение, цилиндрические координаты, эффект железобетона, линейные и угловые деформации, обобщенная гипотеза

Заявление о конфликте интересов. Авторы заявляют об отсутствии конфликта интересов.

Вклад авторов. *Колчунов Вл.И.* — научное руководство; концепция исследования; развитие методологии; написание исходного текста; итоговые выводы. *Федорова Н.В.* — участие в разработке материала; доработка текста; итоговые выводы. *Ильющенко Т.А.* — участие в разработке материала, обработка и редактирование материала.

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1. Introduction

Mechanics of reinforced concrete is fundamental for ensuring mechanical safety of buildings and structures in the conditions of new challenges of man-made, natural and terrorist nature. One of the key and extremely controversial problems of the modern theory of reinforced concrete is the problem of crack opening. For solving this problem, in the last two or three decades, considerable amount of information on the mechanics of deformation and cracking in reinforced concrete has been accumulated worldwide [1–9], including such regulatory documents introduced into the design practice as ACI Committee 318-14, EN 1992-1-2: 2004, SP 5.03.01-2020, SP 63.13330.2018[1], etc. A lot of models, associated with a large number of theoretical and experimental investigations, have been developed over this period, among which the studies of Russian [10–14] and foreign [15–21] scientists can be mentioned. In the last two decades, in the framework of such concept and on a common methodological basis, studies on this problem are conducted also under Vl.I. Kolchunov's supervision [22–27]. In this regard, this paper discusses modelling of the bond between reinforcement and concrete, taking into account physical nonlinearity and the presence of transverse cracks, and using a two-level approach: at the first level, the entire reinforced concrete element is analyzed as a bar, while at the second level, the volumetric stress-strain state of the near-reinforcement zone is considered using a number of parameters obtained from the first-level model.

2. Methodology

Combined action of concrete and reinforcement in a reinforced concrete element is ensured by the near-reinforcement zone. This is a local zone of concrete directly adjacent to the lateral surface of the reinforcing bar and ending (according to the Saint-Venant principle) at some radial distance *tb* (Figire 1). Therefore, the problem will be solved using cylindrical coordinates. The positive directions of these coordinates are given in Figure 1.

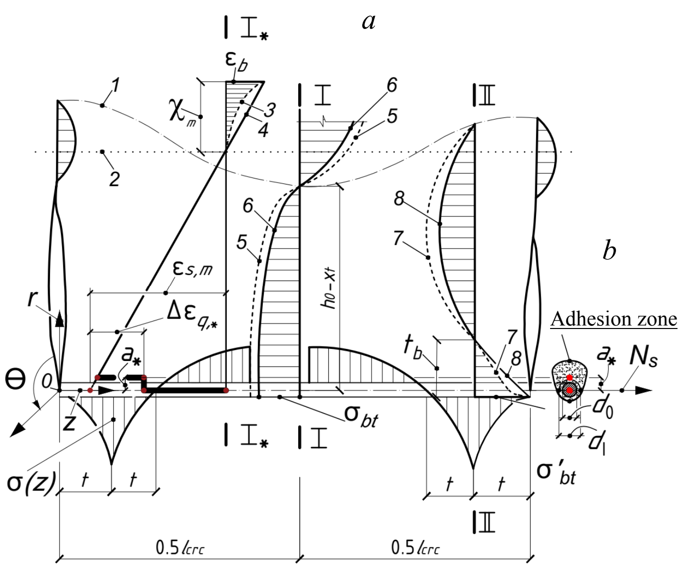


Diagram of section I–I between cracks *e* and section II–II in the stress diagram;

sections I *–I* between cracks in the average relative deformation diagram;

1 — physical neutral axis of reinforced concrete,

2 — average neutral axis of reinforced concrete

Figure 1. Regarding the analysis of experimental and numerical studies in solving the problem of determining the stress-strain state of the near-reinforcement zone:
a — characteristic sections; *b* — in the zone of adhesion of concrete to reinforcement
S o u r c e: made by Vl.I. Kolchunov

[1] See: ACI Committee 318-14. *Building Code Requirements for Structural Concrete and Commentary*. Farmington Hills, Mich: American Concrete Institute, 2014; 519 p.; EN 1992-1-2: 2004. Eurocode 2: Design of concrete structures — Part 1-1: *General rules and rules for buildings, 2004. 225 p. SP 5.03.01-2020. Concrete and Reinforced Concrete Structures*. Minsk; 2020. 236 p.; (In Russ.) SP 63.13330.2018. *Concrete and Reinforced Concrete Structures. General Provisions*. Moscow: Minstroy; 2018.152 p. (In Russ.)

Theoretical solution to the problem under consideration was preceded by a number of experimental and numerical studies [10–27]. As a result of the experimental studies (using continuous chains of strain gauges for measuring deformations), a qualitative picture of concrete strain along the *Z*-axis has been obtained (Figure 1).

The results of such studies allowed not only to find the qualitative nature of the strain distribution in concrete, reinforcement and their mutual displacements in the near-reinforcement zone along the *Z*-axis, but also to obtain a theoretical solution for determining a number of strength parameters of a reinforced concrete bar element, taking into account physical nonlinearity and the presence of transverse cracks.

Thus, the first aspect of the following solution of the volumetric stress-strain state in the near reinforcement zone is that a multilevel analysis approach is used here: at the first level, the entire reinforced concrete element is analysed as a bar; at the second level, the volumetric stress-strain state of the near reinforcement zone is considered using a number of parameters obtained from the first level model. In turn, the output integral parameters of the second-level model influence the parameters of the first-level model at the next iteration stage. Such an approach is possible, for example, when using parameter ψ_s, traditional for reinforced concrete, which allows to update the average value of the parameters of the first-level model without changing the model.

The second important aspect of the proposed solution is that after the formation of cracks (both transverse and longitudinal), fracture mechanics hypotheses are involved in the analysis.

3. Results and Discussion

The solution will be derived taking into account the physical nonlinearity and particular nature of concrete, being different resistance to tension and compression. Relationship diagrams σ_{bi} — ε_{bi}, $\mu(\lambda)$ — ε_{bi}, σ_{bt} — ω are used in the analysis.

The projection of the σ_{bi} — ε_{bi} diagram onto axes τ_{rz} — γ_r allows to establish the relationship between shear stresses and shear strains or, when local shearing is introduced, between the relative mutual displacements of reinforcement and concrete ε_q. It is important to emphasize that, since the stresses in the concrete of the near-reinforcement zone adjacent to the crack are compressive in the cross-section of a reinforced concrete element (see Figure 1), the shear stresses are determined by the upper branch of the strain diagram. This will have a significant effect on the maximum value of shear stresses. Thus, the zones of maximum shear bond stresses are concentrated in proximity to transverse cracks. Therefore, on one hand, the greater the number of transverse cracks crossing the reinforcing bar, the better the bond between the reinforcing bar and concrete in the near-reinforcement zones (before yielding of the reinforcement or formation of radial cracks). On the other hand, as the load increases, the adhesion of concrete and reinforcement in the region between cracks (at $z_1 = 0{,}5l_{\mathrm{crc}}$) first increases, and then, due to the specificity of deformation of the concrete matrix (Figures 1, 2), begins to decrease.

Firstly, this is associated with the fact that in these regions, the stresses in the concrete of the near-reinforcement zone are tensile in the cross-section of the reinforced concrete element. This affects both the current value of the shear stress and its possible maximum.

Secondly, as transverse cracks appear, the area and magnitude of tensile stresses between cracks decrease. In view of the above, special attention is required for the highlighted sections (see Figures 1, 2) located at distances $z = t$ and $z = z_1$. An additional assumption concerning the near-reinforcement zones adjacent to sections I–I and II–II, where it is necessary to determine the stress-strain state of concrete, is taken. The strain diagrams of concrete in tension in section I–I and concrete in compression in the near-reinforcement zone in section II–II are assumed to be linear. This assumption is consistent with the ideas about the resistance of cross-sections of bar elements and is confirmed by numerous experiments. Stresses σ_{bt} and σ'_{bt} (see Figure 1) in a reinforced concrete bar element are determined from the conditions of equilibrium of moments in sections I–I and II–II with respect to the point of application of the compressed concrete resultant force. The values of strain $\varepsilon_b(z)$ in sections I–I and II–II are determined taking into account the strain incompatibility of concrete and reinforcement. They are denoted as $\varepsilon_{z,\mathrm{I}}$ and $\varepsilon_{z,\mathrm{II}}$ respectively. Using the σ_{bi} — ε_{bi} diagram for section I–I, stress $\sigma_{z,\mathrm{I}}(r)$ is found. Given that the strains and stresses along the z-axis are considered, the diagrams are projected onto this axis. Since it is known that the curvilinear segments of the diagram are described by a square parabola and the coordinates of the end points of these parabolas ($\varepsilon_{z,\mathrm{I}}$ and σ_{bt}, $\varepsilon_{z,\mathrm{II}}$ and σ_b) are specified, then the angle between the considered axes is needed only to determine the initial modulus of the $\sigma_{b,z}$ — $\varepsilon_{b,z}$ diagram:

$$E_{b,z} = \varphi E_b, \tag{1}$$

where φ is the coefficient accounting for combined stress state and the presence of shear stresses along axis z[2].

In this case, the value of the shear bond stress and normal stress $\sigma_{b,z}$ is determined at a load of 0.2 of the cracking moment M_{crc}. After substituting the expression for determining the strain in section I–I into the expression for determining the stress in the same section, the following equation is obtained:

$$\sigma_{z,\mathrm{I}}(r) = k_1 \varepsilon_{z,\mathrm{I}}^2 D^2 \left(1 - \frac{r}{t_1}\right)^2 + \varphi E_b \varepsilon_{z,\mathrm{I}} D \left(1 - \frac{r}{t_1}\right), \tag{2}$$

where $k_1 = \dfrac{\sigma_{bt} - \varphi E_b \varepsilon_{z,z}}{\varepsilon_{z,\mathrm{I}}^2}$; $D = -\varepsilon_{z,\mathrm{I}} + \Delta\varepsilon_q$; $t_1 = h_0 - x_t - a_*$.

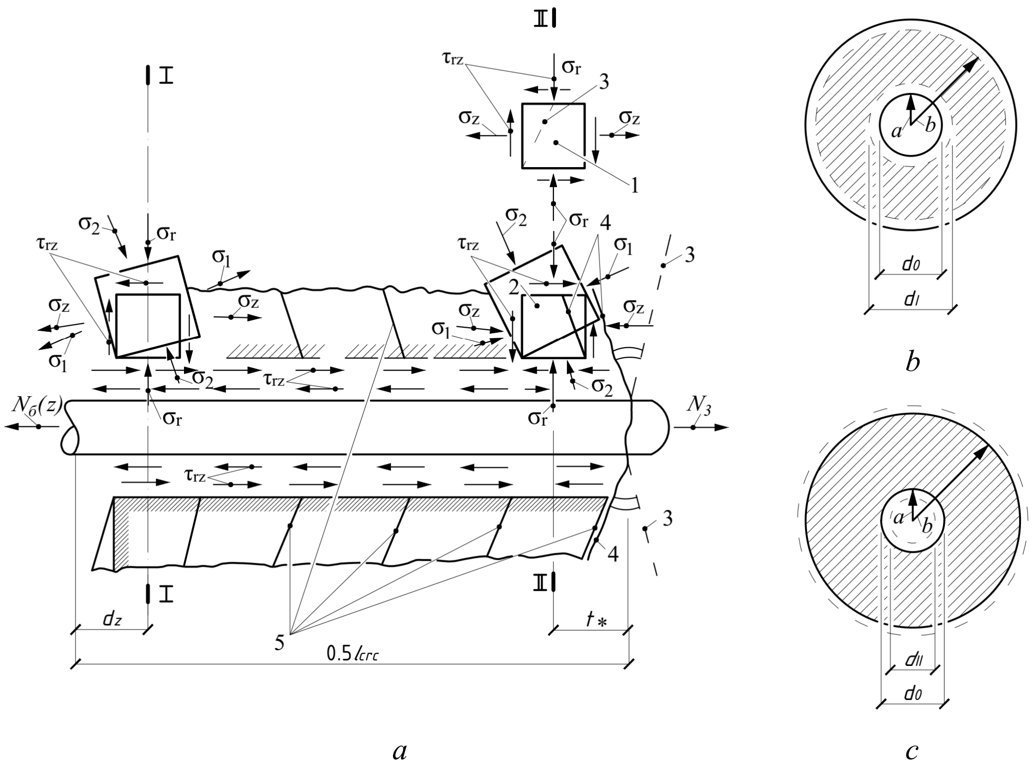


Figure 2. Diagram of the stress state:
a — in the near-reinforcement zone, *b, c* — deformation of the concrete matrix in sections I–I and II–II respectively:
1, *2* — stress-strain state before and after crack formation, respectively;
3, *4* — direction of microcracks and macrocracks, respectively; *5* — deformation of concrete
S o u r c e: made by Vl.I. Kolchunov

[2] Veryuzhsky Yu.V., Kolchunov V.I. *Methods of reinforced concrete mechanics*: textbook. Kyiv: NAU Book Publ.; 2005. (In Russ.)

The fundamental equations for the near-reinforcement zone, with respect to the case when only transverse cracks are present and longitudinal and radial cracks have not been developed yet, are now considered.

Taking into account that the problem is axisymmetric, the values of $\dfrac{\partial\sigma_\theta}{\partial\theta,\ \tau_{\theta r}}$ and τ_{0z} are equal to zero and the equations of equilibrium in cylindrical coordinates take the form

$$\frac{\partial\tau_{rz}}{\partial r}+\frac{\partial\sigma_z}{\partial z}+\frac{\tau_{zr}}{r}=0; \tag{3}$$

$$\frac{\partial\sigma_r}{\partial r}+\frac{\partial\tau_{rz}}{\partial z}+\frac{\sigma_r-\sigma_\theta}{r}=0. \tag{4}$$

The Couchy geometrical relations will also simplify, as displacement components υ and strains $\gamma_{r\theta}$, $\gamma_{\theta z}$ are also equal to zero due to symmetry. The rest of the strain components have the form

$$\varepsilon_r=\frac{\partial u}{\partial r};\ \varepsilon_\theta=\frac{u}{r};\ \varepsilon_z=\frac{\partial w}{\partial z};\ \gamma_{rz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}. \tag{5}$$

The constitutive equations for the concrete matrix, which is modelled as an elastoplastic isotropic body between the transverse cracks, are expressed as

$$\varepsilon_r=\frac{1}{E(\lambda)}\left[\sigma_r-\mu(\lambda)(\sigma_z+\sigma_\theta)\right]; \tag{6}$$

$$\gamma_{rz}=\frac{\tau_{rz}}{\zeta(\lambda)}, \tag{7}$$

where $E(\lambda)$, $\zeta(\lambda)$ and $\mu(\lambda)$ are respectively the elastoplastic deformation moduli and the Poisson's ratio for concrete.

It should be emphasized that for expression (2) in the second parentheses, the differentiation with respect to z is first performed and then it is evaluated at $z = z_1$.

Then for the near-reinforcement zone between transverse cracks, equation (3), considering expression (2), will have the form

$$\frac{\partial\tau_{rz}}{\partial r}+A_{1,\mathrm{I}}D^2\left(1-\frac{r}{t_1}\right)^2+A_{2,\mathrm{I}}D\left(1-\frac{r}{t_1}\right)+\frac{\tau_{rz}}{r}=0. \tag{8}$$

The following notation can be introduced:

$$\left.\left(\frac{\partial k_1}{\partial z}\varepsilon_z^2+2k_1\varepsilon_z\frac{\partial\varepsilon_z}{\partial z}\right)\right|_{z=z_1}=A_{1,\mathrm{I}}\ ;\quad \left.\varphi E_b\frac{\partial\varepsilon_z}{\partial z}\right|_{z=z_1}=A_{2,\mathrm{I}}.;\ t_1=h_0-x_t-a_*.$$

The solution to differential equation (8) is adopted in the form of a product of two functions of r:

$$\tau_{rz}=u(r)\upsilon(r). \tag{9}$$

The following is obtained after differentiation of both sides of equality (9) and their substitution into equation (8):

$$u\left(\frac{\partial \upsilon}{\partial r}+\frac{\upsilon}{r}\right)+\upsilon\frac{\partial u}{\partial r}=-A_{1,\mathrm{I}}D^2\left(1-\frac{r}{t_1}\right)^2-A_{2,\mathrm{I}}D\left(1-\frac{r}{t_1}\right). \tag{10}$$

Function υ is selected such that the expression in the parentheses of the left-hand side of equation (10) is equal to zero. Then, the following is obtain after separation of variables in this equation:

$$\frac{\partial \upsilon}{\upsilon}=-\frac{\partial r}{r}, \tag{11}$$

and after integration: $\upsilon = r^{-1}$.

By substituting the determined value of $\upsilon(r)$ into equation (10) and performing algebraic transformations and integration, the following is obtained:

$$u(r)=A_{1,I}\left(-\frac{D^2r^2}{2}+\frac{2D^2r^3}{3t_1}-\frac{D^2r^4}{4t_1^2}\right)+A_{2,\mathrm{I}}\left(-\frac{Dr^2}{2}+\frac{Dr^3}{3t_1}\right)+C_1. \tag{12}$$

After substitution of $\upsilon(r)$ and $u(r)$ into equation (9):

$$\tau_{rz}=r^3A_{3,\mathrm{I}}+r^2A_{4,\mathrm{I}}+rA_{5,\mathrm{I}}+\frac{C_1}{r}. \tag{13}$$

Here

$$A_{3,\mathrm{I}}=-\frac{A_{1,\mathrm{I}}}{4t_1^2};\;\; A_{4,\mathrm{I}}=-\frac{2D^2}{3t_1}+\frac{A_{2,\mathrm{I}}D}{3t_1};\;\; A_{5,\mathrm{I}}=A_{1,\mathrm{I}}\left(-\frac{D^2}{2}\right)+A_{2,\mathrm{I}}\left(-\frac{D}{2}\right). \tag{14}$$

Integration constant C_1 is determined from the condition that $\tau_{rz} = \tau_{r,\mathrm{I}}$ at $r = a_*$, Here, $\tau_{r,\mathrm{I}}$ is the shear stress in section I–I, which is known from the analysis of a reinforced concrete bar element assuming strain incompatibility of concrete and reinforcement[3].

Then

$$C_1=A_{1,\mathrm{I}}\frac{a_*^2}{(a_*-1)}\left(D^2a_*-\frac{D^2a_*^2}{t_1}+\frac{D^2a_*^3}{3t_1^2}\right)+A_{2,\mathrm{I}}\frac{a_*^2}{(r-1)}\left(Da_*-\frac{Da_*^2}{t_1}\right)-a_*^4A_{3,\mathrm{I}}-a_*^3A_{4,\mathrm{I}}-a_*^2A_{5,\mathrm{I}}. \tag{15}$$

In the near-reinforcement zone adjacent to the transverse crack (section II–II), equation (13) will have a similar form. In this case

$$\left.\left(\frac{\partial k_2}{\partial z}\varepsilon_z^2+k_2 2\varepsilon_z\frac{\partial \varepsilon_z}{\partial z}\right)\right|_{z=z_{\mathrm{II}}}=A_{1,\mathrm{II}};\;\; \left.\varphi E_b\frac{\partial \varepsilon_z}{\partial z}\right|_{z=z_{\mathrm{II}}}=A_{2,\mathrm{II}}. \tag{16}$$

Parameters $A_{3,\mathrm{II}}$–$A_{5,\mathrm{II}}$ and C_2 differ in the fact that in relationships (14) and (15) t_b needs to be substituted instead of t_1.

[3] Veryuzhsky Yu.V., Kolchunov V.I. *Methods of reinforced concrete mechanics*: textbook. Kyiv: NAU Book Publ.; 2005. (In Russ.)

Then, having stress components τ_{rz} calculated, the second differential equilibrium equation (4) can be considered. It is easier to solve the problem in terms of stresses. From the equation for determining the strains in section I–I it follows that

$$\sigma_\theta = \frac{1}{\mu(\lambda)}\left(\sigma_z - \varepsilon_z E(\lambda)\right) - \sigma_r. \tag{17}$$

Substituting expressions (17) and (13) into equation (4) results in a differential equation, which after algebraic transformations will take the following form:

$$\frac{\partial \sigma_r}{\partial r} + r^3 B_1 + r^2 B_2 + r B_{10} + \frac{1}{r} B_{11} + \frac{2\sigma_r}{r} = B_{12}. \tag{18}$$

Here the values of parameters $B_1 - B_7$ are determined according to the work of Y.V. Veryuzhsky, V.I. Kolchunov[3].

The solution of differential equation (18) is also adopted in the form of a product of two functions, which after a number of similar transformations is reduced to the following expression:

$$\sigma_r = -\frac{r^4 B_1}{4} - \frac{r^3 B_2}{5} - \frac{r^2 B_{10}}{4} - \frac{B_{11}}{2} + \frac{r B_{12}}{3} + \frac{C_2}{r^2}. \tag{19}$$

Integration constant C_2 is determined from the condition that $\sigma_r = 0$ at $r = b_*$:

$$C_2 = \frac{b_*^6 B_1}{6} + \frac{b_*^5 B_2}{5} + \frac{b_*^4 B_{10}}{4} - \frac{b_*^3 B_{12}}{3} + \frac{b_*^2 B_{11}}{2}. \tag{20}$$

Here, parameter b_* is determined from the condition that at $r = b_*$ local stresses τ_{rz} in the zone adjacent to the reinforcement practically decay, i.e. their values approach zero. Then it follows from equation (13) that

$$b_*^3 A_{3,\mathrm{I}} + b_*^2 A_{4,\mathrm{I}} + b_* A_{5,\mathrm{I}} + \frac{C_1}{b_*} = 0. \tag{21}$$

Relationship (21) can be used to determine parameter b_*. Taking into account that the rate of change of this function is quite substantial, even small changes of b_* lead to significant changes of stress τ_{rz}. Numerical studies show that at $b_* = 3\ldots4a$, the values of τ_{rz} can be considered as approaching zero. Moreover, at $b_* > 4a$, the outer radius of the near-reinforcement zone can be considered infinitely large (with an error of less than 6%). In this case, the solution is no longer related to the shape of the outer contour. Thus, formulas (13), (17), (19) characterize the stress distribution for the near-reinforcement zone with any shape of the outer contour of the cross-section of a reinforced concrete element.

Knowing stress components σ_z, σ_r, σ_θ, τ_{rz}, the strain components are determined using formulas (6), (7). Then, the displacement components are found from the Cauchy relationship (5):

$$u = \int \varepsilon_r dr + f_1(z); \tag{22}$$

$$w = \int \varepsilon_z dz + f_2(r); \tag{23}$$

$$\gamma_{rz} = \frac{\tau_{rz}}{G(\lambda)} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}. \tag{24}$$

Here ε_r, ε_z are determined according to equation (6), and τ_{rz} is determined according to equation (13). Substituting the values of σ_z, σ_θ and σ_r from relationships (2), (17) and (19) into equation (6) and performing algebraic transformations taking into account (22) and (23), the values of displacements u, w are obtained. After substituting the latter and the stresses from equation (19) into equation (24), considering the integration of the equations, the following is obtained:

$$u = D_1 + C_3 z + C_5; \tag{25}$$

$$w = D\left(1 - \frac{r}{t_1}\right)\left(\int \varepsilon_{z,1} dz\right)\Big| z - z, \mathrm{I} + \eta_1 \frac{r^6}{6} + \eta_2 \frac{r^5}{5} + \eta_3 \frac{r^4}{4} +$$

$$+\eta_4 \frac{(r-a)^4}{4} + \eta_5 \frac{r^3}{3} + \eta_6 \frac{r^2}{2} - 2C_1 \frac{1+\mu(\lambda)}{\mathrm{E}(\lambda)} \ln r - C_4 r + C_6. \tag{26}$$

Here, the values of $\eta_1 - \eta_8$, $C_1 - C_6$, D_1, E_1, H_1 are determined according to the work of Y.V. Veryuzhsky, V.I. Kolchunov[4].

In case of the near-reinforcement zone adjacent to the transverse crack (section II–II), equations (13)–(26) will have similar form. In this respect, equation (16) is used, and values $\varepsilon_{z,\mathrm{II}}$, $A_{1,\mathrm{II}}$, t_b, $\tau_{z,\mathrm{II}}$, $\varepsilon_{q,\mathrm{II}}$ are substituted in these equations instead of $\varepsilon_{z,\mathrm{I}}$, $A_{1,I}$, t_1, $\tau_{z,I}$, $\varepsilon_{q,I}$ respectively.

In all of the above formulas, deformation modulus $E(\lambda)$ and coefficient $\mu(\lambda)$ are determined from the $\sigma_{bi} - \varepsilon_{bi}$ and μ diagrams. The strains are determined by the formula

$$\varepsilon_{bi} = \frac{\sqrt{2}}{2\left[1+\mu(\lambda)\right]} \sqrt{(\varepsilon_z - \varepsilon_r)^2 - (\varepsilon_r - \varepsilon_e)^2 + (\varepsilon_e - \varepsilon_z)^2 + \frac{3}{2}\gamma_{rz}^2}. \tag{27}$$

Stress σ_{bi} is determined as a function of concrete strain ε_{bi}. Then

$$E(\lambda) = \frac{\sigma_{bi}}{\varepsilon_{bi}}. \tag{28}$$

A more complex relationship is used for $\mu(\lambda)$:

- at $\varepsilon_{bi} < \varepsilon_{\mathrm{crc}.c}$, the value of $\mu(\lambda)$ is equal to 0.2;
- at $\varepsilon_{\mathrm{crc}.c} < \varepsilon_{bi} < \varepsilon_{\mathrm{crc}.v}$, the value of $\mu(\lambda)$ is calculated according to formula

$$\mu(\lambda) = 0{,}2 + 0{,}3 \frac{\varepsilon_{bi} - \varepsilon_{\mathrm{crc},c}}{\varepsilon_{\mathrm{crc},v} - \varepsilon_{\mathrm{crc},c}}, \tag{29}$$

- at $\varepsilon_{\mathrm{crc}.v} < \varepsilon_{bi} < \varepsilon_v$ the value of $\mu(\lambda)$ is equal to 0.5;
- at $\varepsilon_v < \varepsilon_{bi} < \varepsilon_{bu}$ the value of $\mu(\lambda)$ is calculated according to formula

$$\mu(\lambda) = 0{,}5 - (0{,}5 - \mu_b) \frac{\varepsilon_{bi} - \varepsilon_v}{\varepsilon_{bu} - \varepsilon_v}. \tag{30}$$

Here, concrete parameters ε_{bi}, ε_{bu} are taken according to the tables, parameters ε_v, μ_b are determined according to diagrams $\sigma_{bi} - \varepsilon_{bi}$.

Considering that the analytical solution for the near-reinforcement zone has been obtained, projecting the $\sigma_{bi} - \varepsilon_{bi}$ diagram onto any axes, for example, onto the $\sigma_z - \varepsilon_z$ axes, does not cause difficulties.

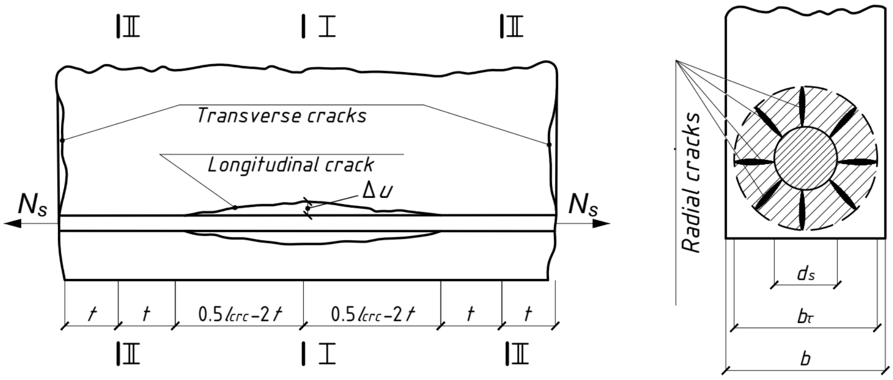
[4] Veryuzhsky Yu.V., Kolchunov V.I. *Methods of reinforced concrete mechanics*: textbook. Kyiv: NAU Book Publ.; 2005. (In Russ.)

Longitudinal cracks (Figure 3, *a*) cancel the adhesion resistance at $\Delta u \le \omega_u$, where ω_u is determined from the relationships of fracture mechanics. The longitudinal crack opening width is calculated by the formula

$$a_{\text{crc},c} = \Delta u = u_b - u_s, \tag{31}$$

where u_b and u_s are the radial displacements of concrete and reinforcement respectively at $r = a$.

In this case, it is possible to take into account the aspects associated with continuity violation at the concrete — reinforcement interface through the boundary conditions.



a *b*

Figure 3. Analysis of longitudinal cracks:
a — in the vicinity of section I–I; *b* — fracture pattern as a result of radial cracks in the vicinity of sections II–II
S o u r c e: made by Vl.I. Kolchunov

Longitudinal cracks along the contact surface of concrete and reinforcement are the most important in the sense of ensuring adhesion of these materials. Displacement u_b is determined by formula (25) at $r = a$. Displacement u_s is determined by formula

$$u_s = \int_0^a \varepsilon_{rs} dr. \tag{32}$$

Here

$$\varepsilon_{rs} \approx \mu_s \frac{\sigma_{si}(z)}{E_s}. \tag{33}$$

More precise values of $\varepsilon_{r,s}$ and u_s can be obtained by using the aspects of the proposed solution for the near-reinforcement zone with respect to the reinforcing bar. However, numerical analysis shows that the influence of stress components σ_{rs} and $\sigma_{\theta s}$ on the value of ε_{rs} is less than 3%. Therefore, it is logical to neglect this influence when solving the problem under consideration in order to simplify the calculations.

Then from (5), the following is obtained after integration:

$$u_s = \frac{\mu_s \sigma_{si}(z)}{E_s}. \tag{34}$$

The integration constant is determined by satisfying the condition that $u_s = 0$ at $r = 0$. Then $C = 0$.

The value of u_s is calculated at $r = a$ according to formula (34).

Longitudinal cracks appear, generally, in the near-reinforcement zones between transverse cracks due to the difference in radial displacements of concrete and reinforcement at the point of their contact (see Fig. 3, *a*). The longitudinal crack profile is close to a triangle with its maximum opening between the transverse cracks (section I–I) and zero opening at a distance of 2*t** from the transverse cracks. In this case, it is assumed that there is no adhesion between concrete and reinforcement in the region of the main longitudinal crack at $\Delta u \geq \omega_u$. However, if the stiffness and strength of the structure is ensured by the bond between concrete and reinforcement in other regions adjacent to transverse cracks (where $\Delta u < \omega_u$), then, taking into account that the presence of longitudinal cracks at the reinforcement surface does not damage the concrete cover, the operation of the structure can be continued.

The proposed analysis approach allows not only to detect transverse cracks, but also longitudinal and radial cracks. Considering that the loading scheme from σ_θ does not cause crack retardation at their apex, the emergence of radial cracks effectively cancels adhesion resistance in this zone, i.e. results in its failure at $a_{\text{crc},r} \geq \omega_u$. The cover layer is destroyed, the reinforcement is bare, so further operation of the structure should be prohibited, even if its resistance due to other regions is not exhausted and it meets the criteria set by the standards for strength and stiffness. Failure from radial cracks (Figure 3, *b*) is characteristic for the near-reinforcement zones adjacent to the transverse crack. Here (depending on the design features), fracture by concrete crushing at the near-reinforcement zone is also possible. In this case, the value of ε_i is calculated by formula (27), where the strain components are calculated at $z = z_{\text{II}}$ and $r = a$.

Note that the proposed solution in terms of stresses has an advantage over a similar solution in terms of displacements. The latter, even within the adopted assumptions, leads to a non-homogeneous second-order differential equation with a large number of particular solutions, which clearly complicates the calculation. An attempt to abandon the use of a multi-level calculation approach complicates the solution to the problem of the volumetric stress-strain state in the zone under consideration so much that it becomes analytically indeterminable. As a result, the solution is possible only by variational methods. Comparing the considered approach with variational methods of solution, which allow to obtain an approximate solution of differential equations with sufficient accuracy for engineering calculations, it can be noted that the proposed solution is first of all simpler. Nevertheless, with respect to the considered problem for reinforced concrete bar elements, this solution is on par in accuracy with the variational methods, when a sufficiently large number of independent functions and terms of the corresponding series are specified in the latter. At the same time, even with such a refined approach, the known methods do not allow to take into account the aspects of continuity violation of the concrete matrix when cracks emerge in it. Hypotheses and methods of fracture mechanics have not yet been properly applied here. The same can be said for the consideration of the strain incompatibility of concrete and reinforcement.

Thus, the differentiated approach to the analysis of the near-reinforcement zone allows to introduce more reasonable criteria for the operation of reinforced concrete structures taking into account not only transverse, but also longitudinal and radial cracks. With that, considering the specific aspects of concrete strength in the near-reinforcement zone (including the strain incompatibility of concrete and reinforcement) brings the analysis substantially closer to the real assessment of the physical phenomena occurring here. At the same time, the proposed methodology preserves the relative simplicity of the calculation and its physical essence, and, consequently, its engineering observability.

4. Conclusion

1. The relevant problem of concrete strength in the near-reinforcement zone has been solved as a problem of the volumetric stress-strain state with "closure" of the output integral parameters of this zone on the framework of the whole reinforced concrete element. In spite of the complexity of such a problem, taking into account the strain compatibility of concrete and reinforcement and the violation of concrete continuity, the solution of the differential equations written in this case is obtained in a closed analytical form.

2. An important aspect of the proposed solution of the problem is the hypotheses of fracture mechanics and the deformation effect of reinforced concrete established by Vl.I. Kolchunov, as well as the consideration of both transverse and longitudinal cracks. At the same time, generalized longitudinal and shear strain hypotheses for cross-section warping, jumps from relative mutual displacements of reinforcement and concrete ε_q, are developed.

3. Physical equations are written for the concrete matrix modeled as an elastoplastic isotropic body between transverse cracks. The solution is obtained considering the physical aspect of concrete, being its different resistance to tension and compression.

4. The developed differentiated approach to the strength analysis of the near-reinforcement region allows, with respect to the reinforcing bar and concrete matrix, to introduce more reasonable criteria for the operation of reinforced concrete structures, taking into account not only transverse, but also longitudinal and radial cracks. Here, stresses σ_θ do not cause crack retardation at their apex, and the emergence of radial cracks effectively cancels the cohesion resistance in this zone and leads to its fracture at $a_{\text{crc},r} \geq \omega_u$. The cover layer is destroyed, the reinforcement is bare, so further operation of the structure should be prohibited, even if its resistance due to other regions is not exhausted and it meets the criteria set by the standards for strength, stiffness and crack resistance for the characteristic failure due to radial cracks.

5. Comparing the proposed method with the variational methods of solution, which allow to obtain only an approximate solution of differential equations, when there is a sufficiently large number of independent functions and terms of the corresponding series, the proposed solution using a multilevel calculation approach does not complicate the volumetric stress-strain state in the considered zone. In such a differentiated approach, hypotheses and methods of fracture mechanics allow to introduce more reasonable criteria for the operation of reinforced concrete structures taking into account not only transverse, but also longitudinal and radial cracks. With that, considering the noted aspects of concrete resistance in the near-reinforcement zone brings the analysis significantly closer to the real assessment of the physical phenomena occurring here. At the same time, the proposed methodology retains the relative simplicity of calculation and engineering observability.

References

1. Bondarenko V.M., Kolchunov V.I. *Computational models of the strength resistance of reinforced concrete.* Moscow: ASV Publ.; 2004. (In Russ.) EDN: QNKPAP

2. Golyshev A.B., Kolchunov V.I. *Resistance of reinforced concrete*. Kyiv: Osnova Publ.; 2009. (In Russ.) EDN: WLZJYN

3. Karpenko N.I. *General models of reinforced concrete mechanics*. Moscow: Stroyizdat Publ.; 1996. (In Russ.)

4. Tureyen A.K., Frosch R.J. Concrete shear strength: Another perspective. *Structural journal*. 2003;100(5):609–615.

5. Wight J.K., MacGregor J.G. *Reinforced concrete mechanics and design*. Pearson Ed.; 2020.

6. Bentz E.C. Empirical Modeling of Cracking in Reinforced Concrete. *ACI Structural Journal*. 2019;116(3):233.

7. Frosch R.J. Another look at cracking and crack control in reinforced concrete. *ACI Structural Journal*. 1999;96(3): 437–442. http://doi.org/10.14359/679

8. Murashev V.I. *Crack resistance, rigidity and strength of reinforced concrete*. Moscow: Mashstroyizdat Publ.; 1950. (In Russ.)

9. Thomas F.G. Cracking in reinforced concrete. *The Structural Engineer.* 1936;14:298–320.

10. Iakovenko I., Kolchunov Vl. The development of fracture mechanics hypotheses applicable to the calculation of reinforced concrete structures for the second group of limit states. *Journal of Applied Engineering Science*. 2017;15(455): 366–375. http://doi.org/10.5937/jaes15-14662

11. Travush V.I., Karpenko N.I., Kolchunov Vl.I., Kaprielov S.S., Demyanov A.I., Bulkin S.A., Moskovtseva V.S. Results of experimental studies of high-strength fiber reinforced concrete beams with round cross-sections under combined bending and torsion. *Structural Mechanics of Engineering Constructions and Buildings*. 2020;16(4):290–297. http://doi.org/ 10.22363/1815-5235-2020-16-4-290-297

12. Klyueva N.V., Yakovenko I.A., Usenko N.V. On the calculation of the opening width of inclined cracks of the third type in composite reinforced concrete structures. *Industrial and civil engineering*. 2014;2:8–11. (In Russ.) EDN: RWGCIT

13. Dem'yanov A.I., Yakovenko I.A., Kolchunov V.I. The development of universal short dual-console element for resistance of reinforced concrete structures under the action torsion with bending. *Izvestiya Vysshikh Uchebnykh Zavedenii, Seriya Teknologiya Tekstil'noi Promyshlennostithis*. 2017;370(4):246–251. (In Russ.) EDN: YLHYQC

14. Travush V.I., Kashevarova G.G., Martirosyan A.S., Kuzminykh V.S. Study of the process of destruction of adhesion bonds during pressing of a rigid reinforcement rod into concrete. Part 1: Experimental studies. *International Journal for Computational Civil and Structural Engineering*. 2016;12(1):140–146. EDN: VZTWHZ

15. Mathern A., Yang J. A practical finite element modeling strategy to capture cracking and crushing behavior of reinforced concrete structures. *Materials*. 2021;14(3):506. http://doi.org/10.3390/ma14030506

16. Golding V. P., Gharineiat Z., Munawar H.S., Ullah F. Crack detection in concrete structures using deep learning. *Sustainability*. 2022;14(13):8117. https://doi.org/10.3390/su14138117

17. Somma G., Vit M., Frappa G., Pauletta M., Pitacco I., Russo G. A new cracking model for concrete ties reinforced with bars having different diameters and bond laws. *Engineering Structures*. 2021;235:112026. https://doi.org/10.1016/j.engstruct.2021.112026

18. Dey A., Valiukas D., Jakubovskis R., Sokolov A., Kaklauskas G. Experimental and numerical investigation of bond-slip behavior of high-strength reinforced concrete at service load. *Materials.* 2021;15(1):293. https://doi.org/10.3390/ma15010293

19. Bado M.F., Casas J.R., Kaklauskas G. Distributed Sensing (DOFS) in Reinforced Concrete members for reinforcement strain monitoring, crack detection and bond-slip calculation. *Engineering Structures.* 2021;226:111385. https://doi.org/10.1016/j.engstruct.2020.111385

20. Kaklauskas G., Sokolov A., Sakalauskas K. Strain compliance crack model for RC beams: primary versus secondary cracks. *Engineering Structures*. 2023;(281):115770. https://doi.org/10.1016/j.engstruct.2023.115770

21. Rimkus A., Cervenka V., Gribniak V., Cervenka J. Uncertainty of the smeared crack model applied to RC beams. *Engineering Fracture Mechanics*. 2020;233:107088. https://doi.org/10.1016/j.engfracmech.2020.107088

22. Kolchunov Vl.I. Generalized hypotheses of warping of linear and angular deformations in reinforced concrete structures under bending with torsion. *Russian Journal of Building Construction and Architecture*. 2023;1(59):9–26. (In Russ.) https://doi.org/10.36622/VSTU.2023.69.1.001

23. Kolchunov V.I. Some problem areas of the modern theory of reinforced concrete and their solutions. *Fundamental, exploratory and applied research of RAASN on scientific support for the development of architecture, urban planning and the construction industry of the Russian Federation in 2021.* 2022:130–141. (In Russ.) EDN: DMBCMT

24. Kolchunov Vl.I. Method of calculation models of resistance for reinforced concrete. *Structural Mechanics of Engineering Constructions and Buildings*. 2023;19(3):261–275. (In Russ.) https://doi.org/10.22363/1815-5235-2023-19-3-261-275

25. Kolchunov Vl.I., Karpenko S.N. Rigidity of reinforced concrete structures under complex resistance. *Russian Journal of Building Construction and Architecture*. 2022;(1):7–20. (In Russ.) https://doi.org/10.36622/VSTU.2022.53.1.001

26. Kolchunov V. The Effect of Reinforced Concrete for Crack Resistance and Rigidity Based on Mechanics of Fracture Under Bending with Torsion. *Modern Problems in Construction: Selected Papers from MPC 2021*. 2022:79–95. https://doi.org/10.1007/978-3-031-12703-8_9

27. Kolchunov Vl.I., Nikulin A.I., Obernikhin D.V. Crack opening width of reinforced concrete structures with trapezoidal cross-section taking into account new resistance effects. *Bulletin of BSTU named after V.G. Shukhov*. 2018;(10):64–73. (In Russ.) https://doi.org/10.12737/article_5bd95a75010906.70019486