Прикладные задачи
нелинейной теории колебаний и волн

Известия высших учебных заведений. Прикладная нелинейная динамика. 2025. Т. 33, № 5
Izvestiya Vysshikh Uchebnykh Zavedeniy. Applied Nonlinear Dynamics. 2025;33(5)

Article DOI: 10.18500/0869-6632-003176

Memristor-based chaotic dynamical model for generating electrocardiogram signal

E. E. Kopets, V. G. Rybin, O. V. Vasilchenko, K. A. Kurtova, T. I. Karimov, A. I. Karimov, D. N. Butusov✉

St. Petersburg Electrotechnical University "LETI", Russia
E-mail: eekopets@etu.ru, vgrybin@etu.ru, ovvasilchenko@etu.ru, kakurtova@stud.etu.ru, tikarimov@etu.ru, aikarimov@etu.ru, ✉dnbutusov@etu.ru
Received 15.04.2025, accepted 29.04.2025, available online 30.04.2025, published 30.09.2025

Abstract. The *purpose* of this study is to create a phenomenological model of the human electrocardiogram based on the McSharry model and to achieve a plausible distribution of interpeak intervals between individual heartbeats. *Methods.* This paper presents an advanced approach to synthetic ECG generation using a modified McSharry model. We used chaotic dynamics instead of conventional pseudorandom number generators to better represent the variability in ECG dynamical parameters, such as interpeak intervals. A fourth-order circuit equation with a memristor is introduced as a chaos generator. By adjusting the parameters of this system, one can vary the range of peak parameters in the synthetic ECG. The proposed ECG generator can be implemented as a computer model or as an analog circuit, depending on the application requirements. *Results.* The experimental investigation of generated synthetic signals with time-domain waveforms, phase portraits, and RR tachograms' analysis demonstrated a good correspondence between the synthetic and real ECGs. It is shown that the modified ECG generation approach provides a reasonably realistic and robust method for simulating synthetic ECG signals. *Conclusion.* The reported solution possesses many possible applications such as the calibration of medical cardiographs, medical education, and machine learning models for ECG analysis.

Keywords: memristor, synthetic electrocardiogram, ECG simulation, nonlinear systems, chaos.

Acknowledgements. This study was supported by the Russian Science Foundation (RSF), project 23-71-01084.

For citation: Kopets EE, Rybin VG, Vasilchenko OV, Kurtova KA, Karimov TI, Karimov AI, Butusov DN. Memristor-based chaotic dynamical model for generating electrocardiogram signal. Izvestiya VUZ. Applied Nonlinear Dynamics. 2025;33(5):691–708. DOI: 10.18500/0869-6632-003176

Научная статья
УДК 530.182

DOI: 10.18500/0869-6632-003176
EDN: UZATFJ

Хаотический генератор электрокардиограммы на основе электронной цепи с мемристором

Е. Е. Копец, В. Г. Рыбин, О. В. Васильченко, К. А. Куртова, Т. И. Каримов, А. И. Каримов, Д. Н. Бутусов✉

Санкт-Петербургский государственный электротехнический университет «ЛЭТИ», Россия
E-mail: eekopets@etu.ru, vgrybin@etu.ru, ovvasilchenko@etu.ru, kakurtova@stud.etu.ru, tikarimov@etu.ru, aikarimov@etu.ru, ✉dnbutusov@etu.ru

Поступила в редакцию 15.04.2025, принята к публикации 29.04.2025, опубликована онлайн 30.04.2025, опубликована 30.09.2025

Аннотация. *Целью* данного исследования является создание феноменологической модели электрокардиограммы человека на основе модели МакШерри и достижение правдоподобного распределения межпиковых интервалов между отдельными сердечными сокращениями. *Методы.* В данной статье представлен передовой подход к генерации синтетической электрокардиограммы с использованием модифицированной модели МакШерри. Мы использовали хаотическую динамику вместо обычных генераторов псевдослучайных чисел для лучшего представления изменчивости динамических параметров электрокардиограммы, таких как межпиковые интервалы. В качестве генератора хаоса представлено уравнение цепи четвертого порядка с мемристором. Регулируя параметры этой системы, можно изменять диапазон пиковых параметров в синтетической электрокардиограмме. Предложенный генератор ЭКГ может быть реализован как компьютерная модель или как аналоговая схема в зависимости от требований приложения. *Результаты.* Экспериментальное исследование сгенерированных синтетических сигналов с временными формами волн, фазовыми портретами и анализом тахограмм продемонстрировало хорошее соответствие между синтетическими и реальными электрокардиограммами. Показано, что модифицированный подход к генерации электрокардиограммы обеспечивает достаточно реалистичный и надежный метод моделирования синтетических сигналов электрокардиограммы. *Заключение.* Представленное решение имеет множество возможных применений, таких как калибровка медицинских кардиографов, медицинское образование и модели машинного обучения для анализа электрокардиограммы.

Ключевые слова: мемристор, синтетическая электрокардиограмма, моделирование электрокардиограммы, нелинейные системы, хаос.

Благодарности. Исследование поддержано Российским научным фондом (РНФ), проект 23-71-01084.

Для цитирования: *Kopets E. E., Rybin V. G., Vasilchenko O. V., Kurtova K. A., Karimov T. I., Karimov A. I., Butusov D. N.* Memristor-based chaotic dynamical model for generating electrocardiogram signal [*Копец Е. Е., Рыбин В. Г., Васильченко О. В., Куртова К. А., Каримов Т. И., Каримов А. И., Бутусов Д. Н.* Хаотический генератор электрокардиограммы на основе электронной цепи с мемристором] // Известия вузов. ПНД. 2025. Т. 33, № 5. С. 691–708. DOI: 10.18500/0869-6632-003176. EDN: UZATFJ

Introduction

Developing mathematical and computer models of physically motivated signal sources is an important task in modern science and technology. Such models can be used to generate augmented data for machine learning [1], better analyze the underlying dynamics of the investigated process [2], be implemented in devices for testing data acquisition hardware, and in many other applications. Developing a model of a real-world signal source is quite a challenging problem, as real systems are often nonlinear, non-stationary, and have high dimensionality, making it difficult to account for all their characteristics in the model. Furthermore, real signals are often noisy, additionally complicating the acquisition of all relevant features for simulation.

Biological systems are among the most complicated systems to model. They are multi-component, and each component depends on the others, but often the structure of their links is only known in general terms. Models of biological data sources can rarely be simplified to stationary systems without a serious loss of generality. Often, they reproduce the dynamics correctly during small periods of time. Also, biological signals are difficult in technical implementation due to their sharp-peaked shape, presence of non-stationary noise, and weakly detectable dependencies of many internal and external factors.

One of the most important biological signals that require modeling is the electrocardiogram (ECG). Electrocardiography is a common method of heart diagnosis in modern cardiology [3]. The ECG records the electrical potentials generated during heart activity. Researchers have proposed various models for generating synthetic biological data for training medical personnel or augmented data for deep learning [4,5]. The ECG model is also useful for filtering real ECG data using the extended Kalman filter [6]. For example, in the study [7] an microcontroller-based circuit was proposed to simulate an electrical signal from the heart. An important application of the ECG model is the test of cardiographs using a synthetic signal. In addition, electronic implementations on the ECG generators can be used for verification and calibration of the other medical instruments, as well as for conducting clinical studies and experiments at laboratories and for training medical personnel. The emergence of new circuit elements, such as memristors, could enable the implementation of many analog biomedical devices with simple and reliable circuitry in the near future [8].

There are several models known to simulate the ECG signal. The well-known model by P. E. McSharry et al. [9] and its modifications are currently in broad use. This model consists of a harmonic oscillator and a modulator that converts the phase of the harmonic oscillation into an ECG waveform [9, 10]. To convert the phase, the modulator uses a sum of Gaussian functions, each corresponding to one peak of the ECG signal. The disadvantage of this model is that it is purely phenomenological, i.e. it reproduces the ECG waveform without accounting for the internal dynamics of the heart. A more complex approach is based on modeling the heart as a system of coupled oscillators. Early examples of this approach include a model proposed by B. van der Pol and J. van der Mark, which used coupled neon lamp oscillators to simulate pacemaker signals [11]. A more detailed version of such a model was described in [12, 13]. However, even this complex approach, which requires selecting a significantly larger number of parameters—many of which can only be indirectly estimated from the ECG signal—provides only a first approximation to the real dynamics. In both cases, small deviations in ECG parameters, such as inter-peak interval variability, are modeled through empirical random distributions or the normal distribution [9], which can be a rather rough approximation. At the same time, as shown three decades ago [14], rhythm variability can be caused by chaotic rather than stochastic dynamics, and more recent work confirms this assumption, at least for some pathological conditions [15, 16].

Therefore, it seems reasonable to propose a phenomenological model since such models are easy to synthesize and set up while providing the necessary variability in signal parameters through an incorporated chaotic oscillator. This would allow for adjustment not only of the dispersion of parameters but also of the degree of their non-linear connection. An additional advantage of such a model would be its simple implementation as an analog electronic circuit.

The main contributions of this work are as follows. First, a modification of the McSharry ECG model is proposed, which uses a memristor-based chaos generator instead of a harmonic oscillator, and allows reproducing the variability of the PQRST complex not by exploiting a random number generator but via mimicking natural dynamical variability. Then, bifurcation diagrams are constructed to indicate chaotic oscillator parameters at which specific dispersions of signal parameters can be obtained. It is shown that the reconstruction of the governing oscillator of a real ECG signal resembles the dynamics of the proposed chaotic oscillator in the plane of phase variables y, z. Second, a novel algorithm for adjusting the ECG shape modulator based on Gaussian functions is proposed, taking into account the unevenness of the phase change in the chaotic oscillator. Thirdly, a possible analog circuit implementation of the proposed ECG generator is suggested.

Detailed investigation of the proposed model shows that it can generate signals with a plausible dispersion of parameters, as confirmed by using the RR tachogram.

1. Related works

One of the most known dynamical models for generating synthetic ECG signals is the model proposed by P. E. McSharry et al. [9]. This versatile model is represented by a system of three ordinary differential equations (1) and can produce a trajectory around an attractive limit cycle in a three-dimensional phase space. Each revolution of the trajectory corresponds to one heartbeat. The model can be transformed into a polar coordinate system, after which the first equation can be omitted. In a study by R. Sameni et al. [6], the authors developed an extended Kalman filter for filtering ECG signals based on the modified two-dimensional model.

The McSharry model is often used to evaluate the performance of noise reduction and waveform detection algorithms, as well as to model pathological signals. In particular, J.T. Shey et al. [17] use the McSharry model to generate synthetic signals with different noise levels to test a real-time ECG signal processing system. D. Kicmerova [18] describes a method for determining the initial parameters of the McSharry model to simulate ECG appearance during arrhythmia.

Another analytical method for generating signals is based on a combination of elementary trigonometric functions and a linear function [19]. Although the proposed model allows precise control of the signal type, it is difficult to use due to the large number of parameters and requires different model variations depending on the desired wave frequency.

In 1928, B. van der Pol and J. van der Mark [11] presented a model of heart rhythms based on an electrical circuit. The generation method involves three coupled oscillatory systems consisting of a resistor, a capacitor, and a nonlinear element – a neon lamp. However, the proposed model was only a rough approximation of how the heart actually functions. Later, S. R. Gois et al. [13], as well as E. Ryzhii and M. Ryzhii [12], developed their own versions of improved models of heart rhythm dynamics using three nonlinear oscillators.

Recently, the possibility of using machine learning to generate synthetic ECG signals has been explored. For example, E. Adib et al. [20] compared two diffusion models, with the WGAN-GP (Wasserstein Generative Adversarial Networks with Gradient Penalty) model generating signals closest to real data. Another study [21] describes an approach to generating synthetic ECGs based on generative adversarial networks to anonymize medical data. Yong Xia et al. [22] compared models for ECG generation based on GAN(Generative Adversarial Networks) and VAE (Variational Auto-Encoder). The study showed that using the VAE model makes the ECG generator more accurate and diverse compared to the GAN model. These methods can create realistic synthetic signals but require a large set of real ECG data and careful processing.

There are other methods for modeling ECG signals. S. Swain et al. [23] proposed an interacting multiple model (IMM)-based scheme that helps dynamically model and estimate the ECG signal. The proposed model can adapt to various morphological representations and does not require user-specific parameters. Another approach described by A. Mishra et al. [24] utilizes the parametric cubic spline approach to construct smooth and continuous curves through a set of control points. The advantages of this approach include smoothness, accuracy, and low computational complexity.

Another technique for acquiring large samples of ECG signals was proposed in a conference paper by K. Vo [25]. The authors used photoplethysmography (PPG) data to construct an ECG signal. To achieve this transformation, a GAN-based network was developed. A similar idea is described by X. Yuan et al. [26], although the method for acquiring ECG information from PPG differs. V. Kuznetsov et al. [27] proposed a method for generating a signal of one cardiac cycle using a variational autoencoder. Reducing the number of features helps lower the computational complexity and model design.

2. Materials and Methods

2.1. Synthetic ECG model. The ECG signal is calculated as the potential difference between two electrodes located on the surface of the human skin. The typical ECG cycle consists of five waves: P, Q, R, S, T (Fig. 1, *a*). The P wave represents the process of atrial myocardial depolarization, the QRS complex reflects ventricular depolarization, and the ST complex and the T wave separately reflect the processes of ventricular myocardial repolarization.

The McSharry model of the ECG is defined by a system (1) of three ordinary differential equations (ODEs). It generates a trajectory in a three-dimensional phase space. The first two equations generate

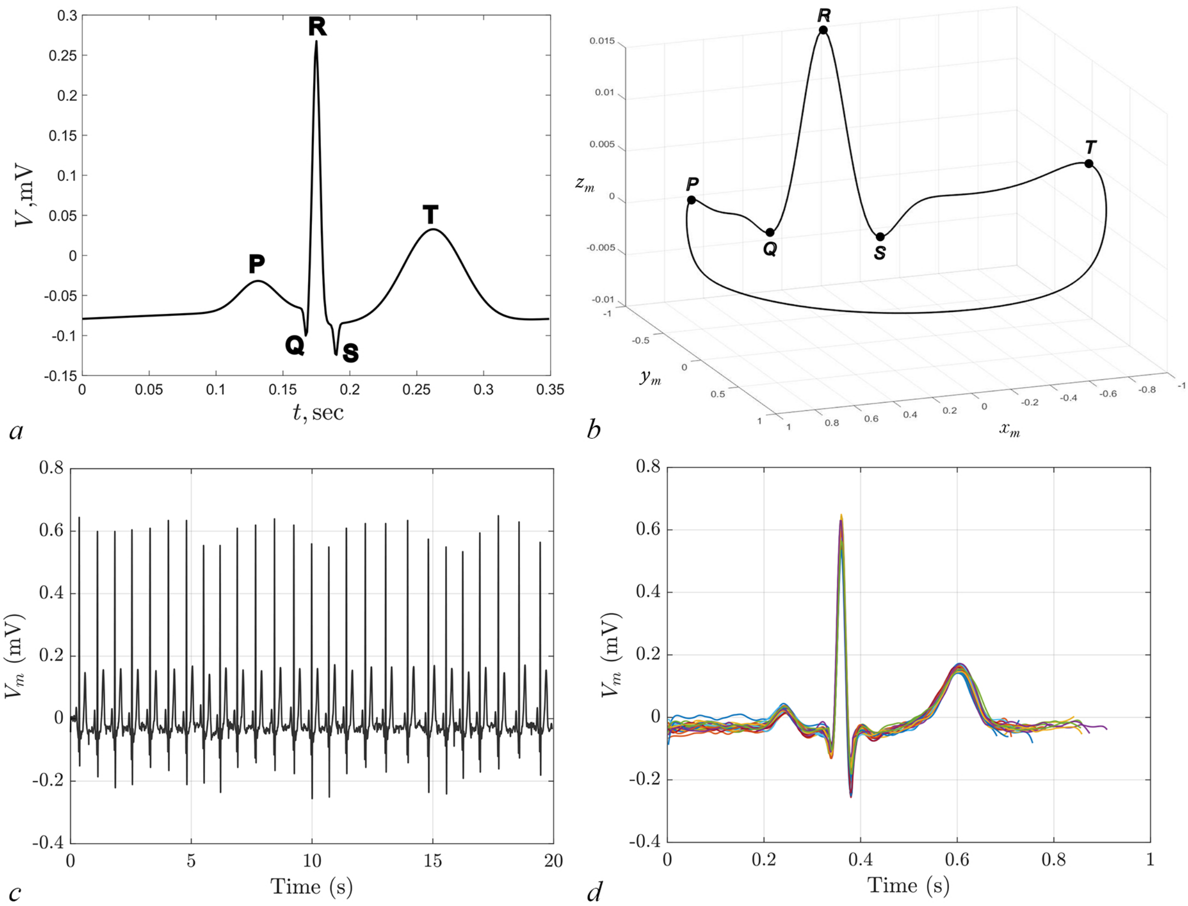

Fig. 1. ECG oscillations related to the R peak: *a* — The one oscillation of ECG, *b* — Phase space of the McSharry model, *c* — ECG signal waveform of real data, *d* — Individual ECG signal samples superimposed on each other at the R peak point (color online)

a limit cycle, and the third equation generates one period of a heartbeat. Each revolution of the limit cycle corresponds to one RR interval or heartbeat.

The model is described by the following ODEs with variables x_m, y_m, z_m (index m here stands for "the McSharry model"):

$$\begin{aligned}
\dot{x}_m &= (1 - \sqrt{{x_m}^2 + {y_m}^2})\, x_m - w\, y_m, \\
\dot{y}_m &= (1 - \sqrt{{x_m}^2 + {y_m}^2})\, y_m + w\, x_m, \\
\dot{z}_m &= - \sum_{i \in \{P,Q,R,S,T\}} a_i \Delta\theta_i \exp\left(-\frac{{\Delta\theta_i}^2}{2{b_i}^2}\right) - (z - z_0),
\end{aligned} \tag{1}$$

where $\Delta\theta_i = (\theta - \theta_i) \mod 2\pi$, $\theta = \mathrm{atan2}(y_m, x_m)$ (phase variable for the ECG model), where $-\pi \leqslant \mathrm{atan2}(y_m, x_m) \leqslant \pi$, and w is the angular velocity of the trajectory equal to $\frac{2\pi}{T}, T$ is the period of one oscillation. The breathing trend is added using the formula:

$$z_0 = A \sin(2\pi f_2 t),$$

where f_2 is the respiratory rate and A is the amplitude. The values of a_i, b_i and θ_i are selected depending on the signal for each of the peaks P, Q, R, S, T. The parameter a_i is directly proportional to the signal amplitude, b_i is the width of the selected Gaussian function, and the parameter θ_i is the position of the peak on the θ axis. Typical values of the parameters are given in the table 1. By changing the parameters of the model, it is possible to achieve any dynamics associated with the positions of the peaks of the PQRST complex. For example, shifting the peak P to the peak T will correspond to the picture of the AB block. In this way, it is possible to expand or narrow the width of any peak or complex using the parameter b_i, shorten or increase the amplitude by varying the parameter a_i and change the positions of the peaks using the parameter θ_i

Table 1. Parameters of the model (1)

	P	Q	R	S	T
θ_i	$-\frac{\pi}{3}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{2}$
a_i	1.2	-5	30	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4

One of the important features calculated from the ECG signal for analysis is the RR interval, which is closely related to the heart rate. Heart rate variability analysis includes the construction of the RR-tachogram (the ratio of the distance of the previous RR interval to the next). In the McSharry model, changes in the length of RR intervals are included by stochastic changes in angular velocity.

2.2. Real data phase space reconstruction. As seen in Figure 1, *d*, the main peaks of the ECG signal are located in approximately the same places, with only the distance between the T and P peaks changing. This means that the period of each individual oscillation will be slightly different, and therefore the frequency of oscillations will also be different. In the model (1), the frequency of oscillations is modulated by adding a normally distributed component with zero mean to the frequency variable w [9].

To display the real ECG signal in the phase space similarly to how it is presented in the model (1), it is necessary to reconstruct the variables x, y from the data. In the model (1), x_m, y_m represent sine and cosine waves with oscillation period T and together form a unit circle. The third variable z_m is modulated by the harmonic motion of this circle. To make the model relate to nonlinear dynamics, we assume that oscillations with different periods T correspond to different diameters of the circle, resembling chaotic motion.

Algorithm 1: Phase space reconstruction of real ECG data

input : $ECGdata, F_s$

output : x_r,y_r

Determine the positions of R peaks in the signal and divide the signal into individual oscillations (Fig. 1). Number of oscillations = L;

for $i \leftarrow 1..L$ **do**

1. Determine the positions of the R peaks, and divide the original signal into segments T_i equal to the distance between the R peaks.
2. Determine the time t_k where $k \in P, Q, R, S, T$ which corresponds to the peaks P, Q, R, S, T respectively;
3. Calculate the angles $\theta_k = 2\pi t_k$;

Add the angles corresponding to the beginning and end of the segment: $\theta_0 = 0$ when $t_0 = 0$ and $\theta_{end} = 2\pi$ when $t_{end} = T_i$

4. Interpolate the obtained values of t_i and θ_i from 0 to 2π:

$$\alpha = \text{interp1}(t_k, \theta_k, [0 : h : T_i]),$$

where $h = 1/F_s$

5. Calculate x and y variables using the formulas:

$$x = T_i \sin\alpha$$

$$y = T_i \cos\alpha$$

6. Add the resulting arrays to x_r and y_r, respectively.

The Algorithm 1 describes the process of finding the first x_r and second y_r variables for real data, where index r stands for "reconstructed". First, the positions of the R peaks in the original signal are determined using the *findpeaks()* function from MATLAB. Next, the signal is trimmed so that it starts from the midpoint of the first R peak. Then, the trimmed signal is divided into individual segments of length T_i, each corresponding to the distance between R peaks. After this, the positions of the corresponding peaks P, Q, R, S, T in each segment are determined and the angles θ_k are calculated. The obtained angle values are interpolated in the range from 0 to 2π with a sampling time step of h. The variables x_r and y_r are calculated using the formulas: $x_r = T_i \sin\alpha$, $y_r = T_i \cos\alpha$. This results in two artificially reconstructed variables representing the time domain behavior of the real signal.

In Fig. 2, *a* and 2, *b* the phase portraits in variables x, y for the model (1) and the real signal are shown, respectively. In this way, we can visually track how variable the signal frequency is.

2.3. The modified ECG model. The main idea of modifying the existing ECG model is to use a chaotic generator instead of a harmonic generator to obtain a system by varying the parameters of which one can achieve different patterns of peak distribution. In this case, it is advisable to use a hyperchaotic system, because the heart rate regulation system is high-dimensional, and classical third-order systems cannot reproduce the broadband chaos observed in heart rate variability.

One of the promising approaches to developing analog chaotic systems, which are simple from the perspective of circuit engineering, is using memristors. According to the theory proposed by L. Chua, a memristor is the fourth basic circuit element, along with a resistor, a capacitor, and an inductor. Currently, many practical implementations of this element are known, such as the product by the

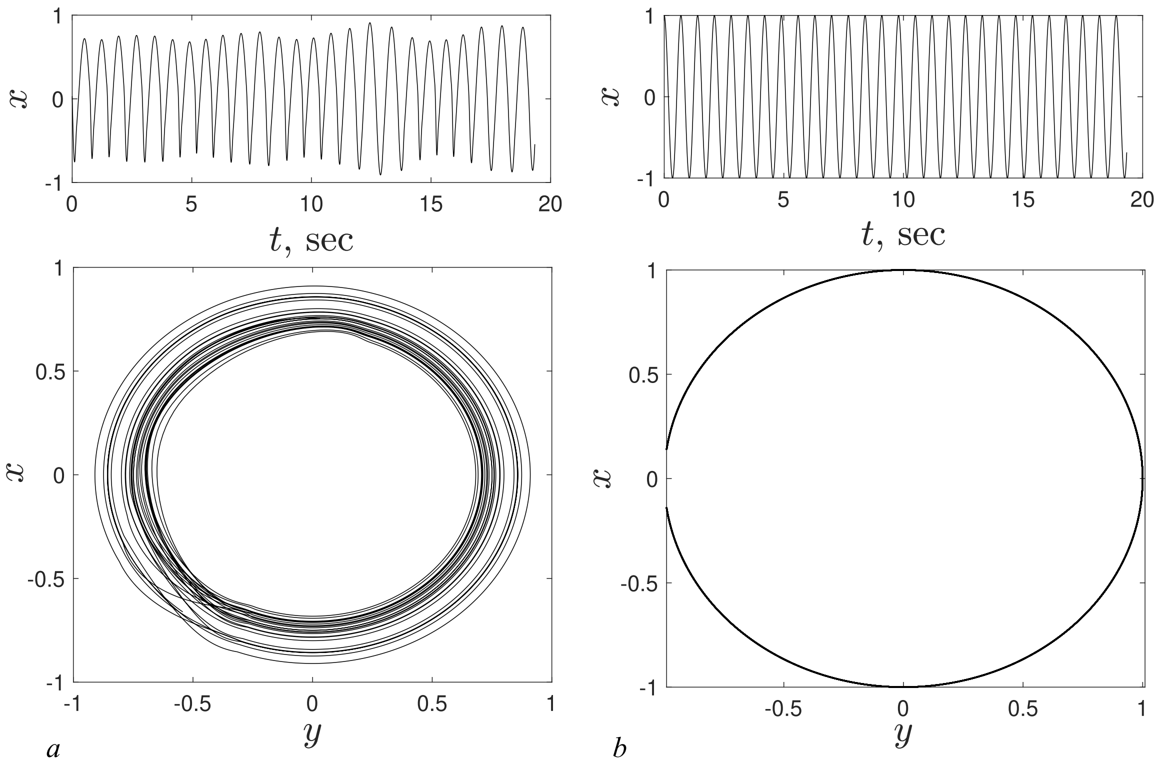

Fig. 2. Variables x, y: *a* — Reconstructed from real data, *b* — Received from model (1)

American company Knowm [28]. Although different real memristors have different parameters, a cubic nonlinear model describing the relationship between the flow φ and the charge q can be used as the first approximation:

$$q(\varphi) = m_0\varphi + \frac{m_1}{3}\varphi^3,$$

where m_0, m_1 — parameters of a cubic parabola. After differentiation, this yields

$$M(\varphi) = \frac{dq}{d\varphi} = m_0 + m_1\varphi^2, \tag{2}$$

where $M(\varphi)$ is the memductance.

The four-dimensional memristive electric circuit from [29] was chosen as the phase θ-generator for the ECG model. The Kirchhoff equations for this circuit can be written as

$$\begin{aligned}
\dot{V}_{C_1} &= \frac{i_L}{C_1} - \frac{V_{C_1}M(\varphi)}{C_1},\\
\dot{V}_{C_2} &= -\frac{i_L}{C_2},\\
\dot{i}_L &= -\left(\frac{V_{C_1}}{L} - \frac{V_{C_2}}{L}\right),\\
\dot{\varphi} &= V_{C_1}.
\end{aligned} \tag{3}$$

The circuit consists of four elements in three branches: capacitor C_1 stands in parallel with series connection of C_2 and L, and memristor M. Device described by (2) is an active magnetic flux-controlled memristor (MFCM), which provides energy flow into the circuit. All elements are reactive, and thus form a 4-th order ODE, where the state variables are: the voltages across capacitors V_{C_1} and V_{C_2}, the current through inductive coil i_L, and the magnetic flux ϕ in MFCM. To form circuit equations using Kirchhoff laws, recall that the current through the memristor is defined as $V_M M(\varphi)$, where V_M is the voltage across the memristor.

From (3), after substituting (2) and replacing $V_{C_1} = x$, $V_{C_2} = y$, $i_L = z$, $\varphi = u$, the following system of ordinary differential equations can be obtained:

$$
\begin{aligned}
\dot{x} &= -az - ax\,m_0 - ax\,m_1 u^2, \\
\dot{y} &= -bz, \\
\dot{z} &= -d(x - y), \\
\dot{u} &= nx,
\end{aligned} \tag{4}
$$

where $a = \frac{1}{C_1} = 3.75, b = \frac{1}{C_2} = 10, d = \frac{1}{L} = 1, n = -1, m_0 = -0.33, m_1 = 0.25$ [29]. Six projections of its attractor onto various planes are presented in Fig. 3. This system was chosen because one of its phase portraits (plane yz, Fig. 3, *c*) represents the distorted circle close to one obtained after reconstruction of the phase plane xy of the McSharry model from data, shown in Fig. 2, *a*.

For this reason, the modified model will use the variables y and z from (4). We define the oscillator phase as the arctangent of two arguments $\theta = \mathrm{atan2}(z, y)$. The phase of one typical period of this chaotic system is distributed unevenly with respect to time, unlike the phase of the conventional harmonic oscillator (1). Therefore, it is necessary to determine new values for the parameters θ_i, a_i, b_i in the formula (1), which will take into account with uneven phase of the chaotic oscillator and will result in an undistorted appearance of the synthetic ECG signal.

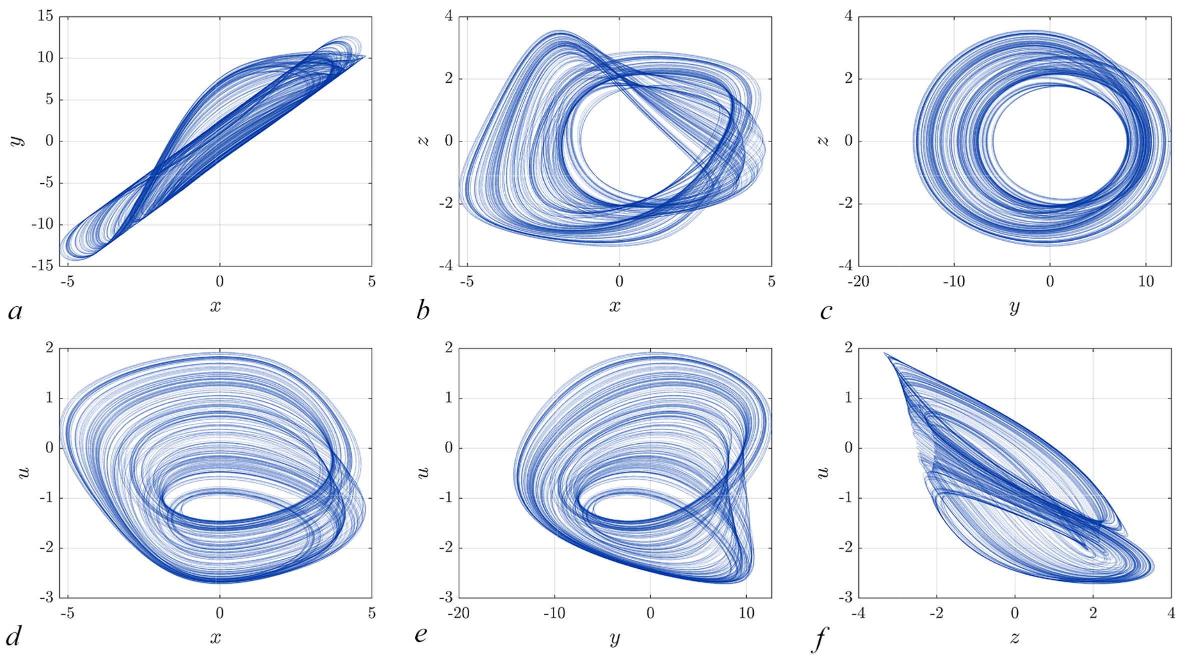

Fig. 3. Phase portraits of a 4-dimensional chaotic memristive system

Algorithm 2: Finding a new formula description for the ECG signal curve

input : y, z, z_m

output : $\dot{z}_n$

1. Find the phase $\theta = \operatorname{atan2}(z, y)$ for an averaged period of the chaotic system;
2. Approximate the curve (θ, z_m) by the sum of five Gaussian functions:

$$f_s(\theta) = \sum_{i=1}^{n=5} a_i \exp\left(-\frac{\theta - \theta_i{}^2}{b_i{}^2}\right),$$

The synthetic ECG signal is a function of $\theta(t)$:

$$z_n(t) = f_s(\theta(t)).$$

3. Differentiate the resulting function $f_s(\theta)$ to obtain the ODE:

$$\begin{cases} \dot{z}_n & = \dfrac{df_s(\theta)}{d\theta}\dfrac{d\theta}{dt}, \\ \dfrac{df_s(\theta)}{d\theta} & = -\displaystyle\sum_{i=1}^{n=5} a_i \frac{2(\theta - \theta_i)}{b_i{}^2} \exp\left(-\frac{(\theta - \theta_i)^2}{b_i{}^2}\right), \\ \dfrac{d\theta}{dt} & = \dfrac{-y^2}{y^2 + z^2}\,\dot{x} + \dfrac{x}{y^2 + z^2}\,\dot{y}. \end{cases}$$

Algorithm 2 describes the process of finding the sum of Gaussian functions with new parameters θ_i, a_i, b_i. At the first stage, the values of the phase θ in the intervals from 0 to 2π for the variables y, z are found for an averaged phase (since the signal is chaotic, every period is unique). Then, the curve (θ, z_m) is approximated by the sum of five Gaussian functions using the `cvtool` tool in MATLAB. The resulting function $f_s(\theta)$ with the selected coefficients a_i, b_i, c_i describes the behavior of the variable z_n. However, for the modified model it is necessary to know the equations for $\dot{z}_n$; therefore, at the next stage, it is necessary to differentiate the resulting function $f_s(\theta)$ to obtain the final ODE.

The resulting ODEs for the proposed modified ECG model with a chaotic oscillator are as follows:

$$\begin{aligned} \dot{x} &= \mu(-az - axm_0 - axm_1u^2), \\ \dot{y} &= \mu(-bz), \\ \dot{z} &= \mu(-d(x - y)), \\ \dot{u} &= \mu(nx), \\ \dot{z}_n &= -\sum_{i \in \{P,Q,R,S,T\}} a_i \frac{2(\theta - \theta_i)}{b_i{}^2} \exp\left(-\frac{(\theta - \theta_i)^2}{b_i{}^2}\right)\dot{\theta} - (z_n - z_0). \end{aligned} \tag{5}$$

where

$$\theta = \operatorname{atan2}(z, y),$$

$$\dot{\theta} = \frac{-z^2}{y^2 + z^2}\,\dot{y} + \frac{y}{y^2 + z^2}\,\dot{z}.$$

The parameter μ is responsible for adjusting the frequency of the chaotic oscillator. The parameter a_i is the amplitude of the i-th peak, θ_i is the position of the peak on the θ axis, and b_i is the width of the peak of the Gaussian function. The parameters a_i, b_i, θ_i for each of the five normal ECG peaks are presented in the table 2. The parameters a, b, d, m_0, m_1, n of the memristor-based chaotic oscillator can be either borrowed from the original publication [29] or selected to achieve the required dynamics, which will be discussed further.

Table 2. Parameters of the modified ECG model

	P	Q	R	S	T
θ_i	-0.4	-0.045	-0.009	0.07096	1.8
a_i	0.0432	-0.03	0.3955	-0.045	-0.055
b_i	0.063	0.032	0.02063	0.016	1.3

Note that in real engineering practice, it may be more convenient to use not an ODE of the form (5), but to synthesize $z_n(t)$ directly as a sum of Gaussian functions. As an analog generator of Gaussian functions, one of the circuits proposed in the review [30] can be used. To convert y, z to phase, instead of $\mathrm{atan2}$, a phase-sensitive detector should be utilized. A possible block diagram of such an ECG generator is shown in Fig. 4.

From Fig. 4, it becomes obvious that this kind of model is a particular simplified case of the Gaussian radial basis function network, which is often used for biological signals simulation and prediction, including the ECG signal [31].

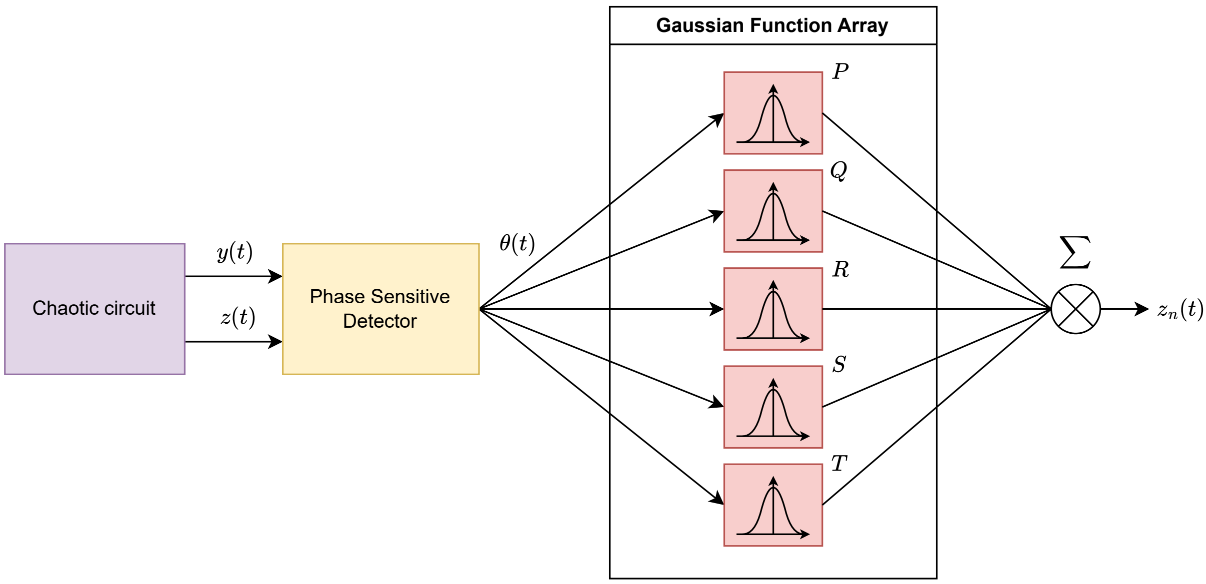

Fig. 4. Structural diagram of an ECG generator based on an analog chaotic circuit with a memristor

3. Results

The generated ECG signal of the proposed model is given in Fig. 5, *a*. The model (5) is described using five ordinary differential equations, but to display the phase portrait, by analogy with the McSharry model, three variables y, z, z_n were used. The ECG signal was modeled in the MATLAB environment using the variable-order method `ode113`, with an integration step $h = 0.0001$ and initial conditions $[0.01, 0.001, 0, 0, 0.1]$. MATLAB codes for generating synthetic ECG, phase portraits and

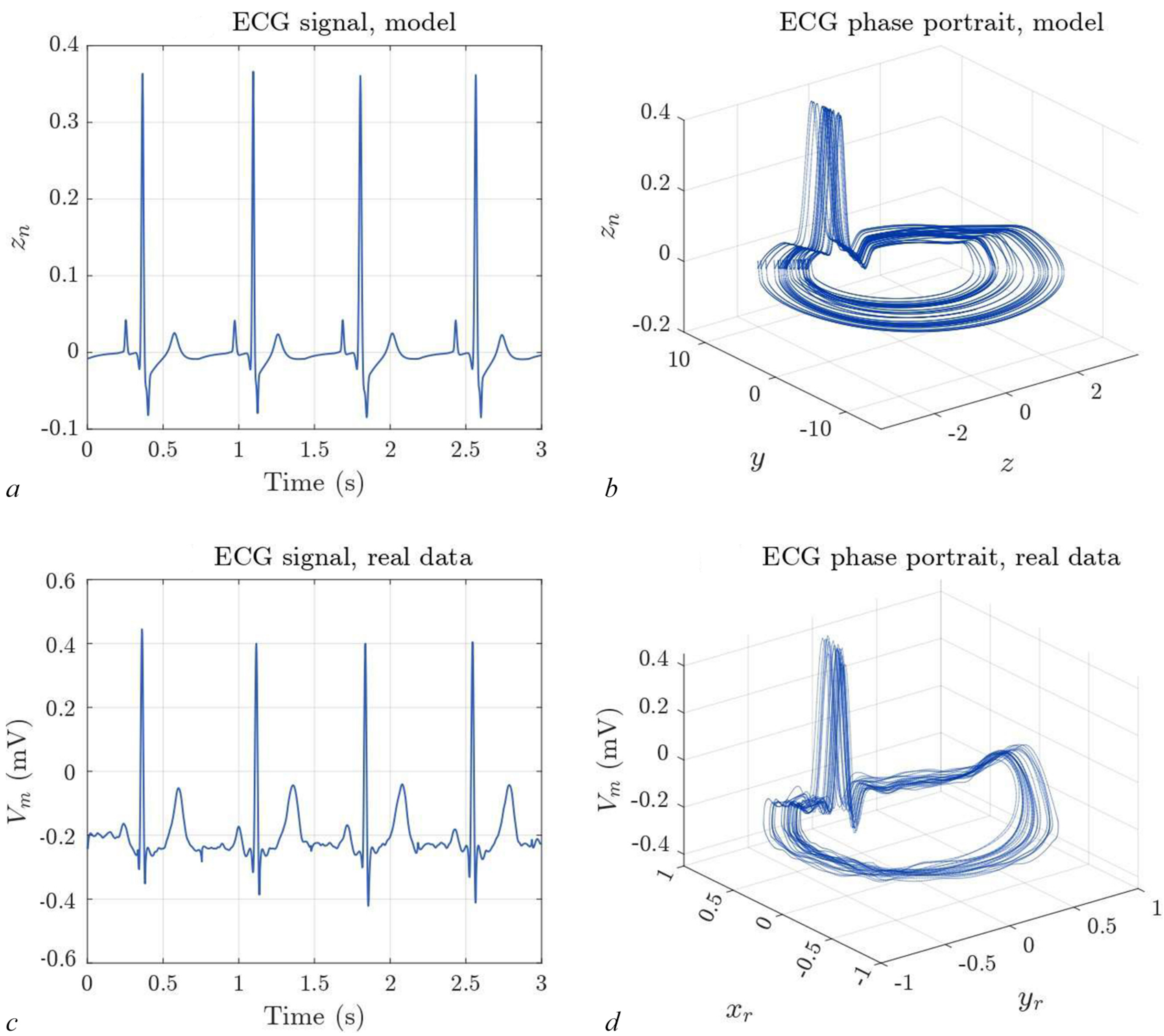

Fig. 5. Synthetic and real data ECG

RR tachograms are available upon request to the authors. Fig. 5 imitate the normal electrocardiogram, however, by changing the parameters a, b, c of the proposed model, it is possible to generate various forms of abnormal electrocardiograms. In this paper, we focused on reproducing the variability of the dynamics of RR peaks, so further in the article we will consider the shape of a normal ECG.

In order to characterize the ECG signal, the RR interval is often used, which is the time between successive R-peaks. The reciprocal of this time interval gives the instantaneous heart rate. One way to visualize the variability of the distance between the R peaks of ECG signal oscillations is to construct RR tachograms; the variability of these RR intervals can reveal a lot about the patient's condition. Fig. 6 shows RR-tachograms for six healthy patients of different ages and two synthetic sequences generated by the proposed model. The data were taken from the ECG-ID database [32,33] and represent 20-second ECG records with post-processing for noise and trend removal. The characteristics of the records used are given in Table 3.

The sampling rate is 500 ticks per second. By varying the parameter μ, the RR tachogram distribution can be shifted along the diagonal: with an increase in the parameter, the distribution will shift down the diagonal, and with a decrease, it will shift up. Two synthetic ECG signals were generated:

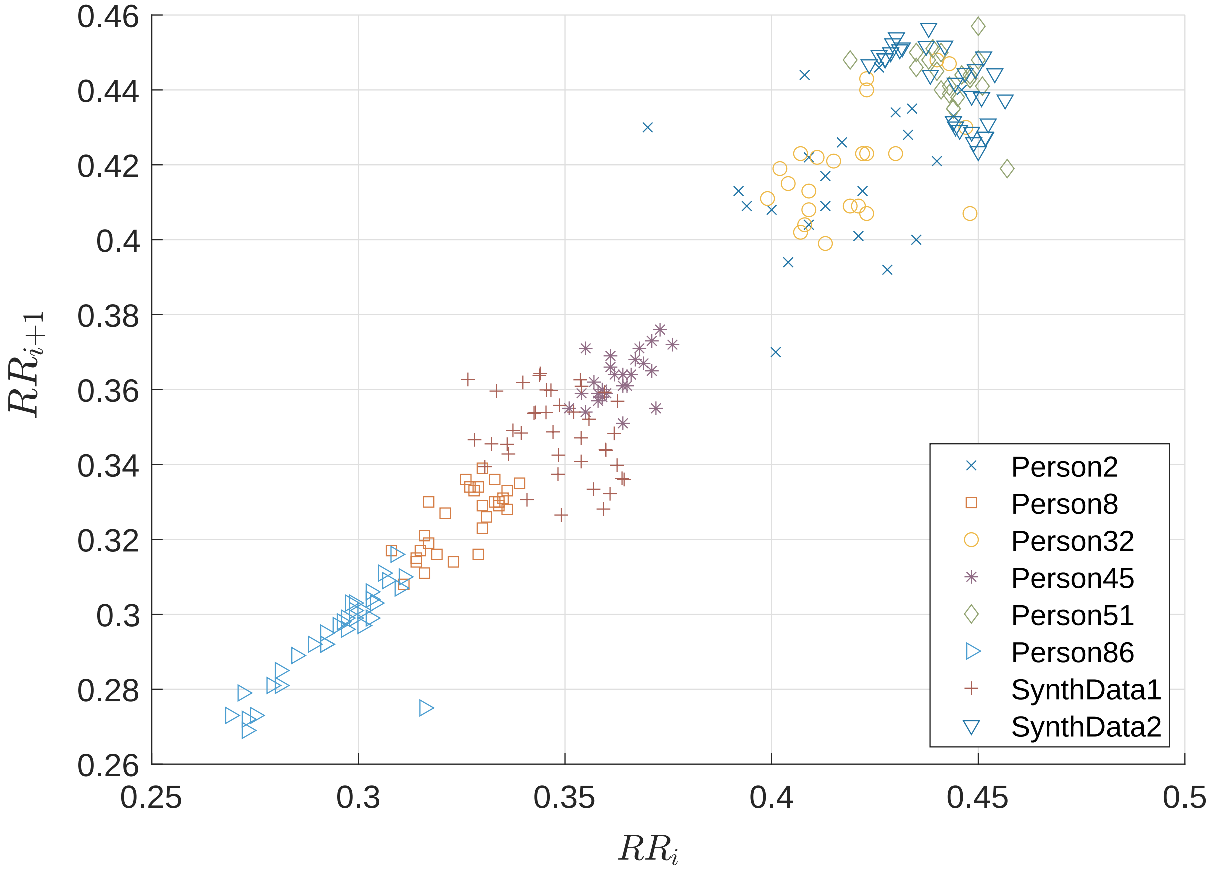

Fig. 6. RR-tachograms of different patients

both with the standard parameters for the model given below the equation (4), but in the first sequence ($Synth_1$), the parameter $a = 4$, and for the second $a = 3.75$. As one can see from Fig. 6, the dispersion of distances between RR peaks can be quite diverse.

In the chaotic ECG model, we propose to vary the parameters a, b, d, n, m_0, m_1 of the model (5) to achieve different distribution patterns of values on RR-tachograms.

Fig. 7 shows bifurcation diagrams for the model parameters, where the x axis represents the value of the parameter under study, and the y axis represents the time distance between RR peaks in the generated ECG signal. For example, increasing the parameter a from the initial value $a = 3.75$ to $a = 4$ will allow us to obtain a significant increase in the dispersion of RR peaks while decreasing it to $a = 3.2$ leads to degeneration of chaos, and the system becomes periodic. Similar behavior can be observed when varying other parameters.

Let us investigate the dynamics of the obtained synthetic ECG signals with simultaneous variation of two parameters. For this purpose, we used the tools previously developed by our research group, described in the work [34]. However, for the current study, we made some changes: we used the values of RR intervals in the synthetic ECG signal instead of local values of maxima and interpeak intervals. The final

Table 3. Parameters of the data records

Data record name	Record number	Age	Sex
Person 2	rec1	25	male
Person 8	rec2	21	female
Person 32	rec4	46	female
Person 45	rec1	75	male
Person 51	rec1	31	male
Person 86	rec2	16	female

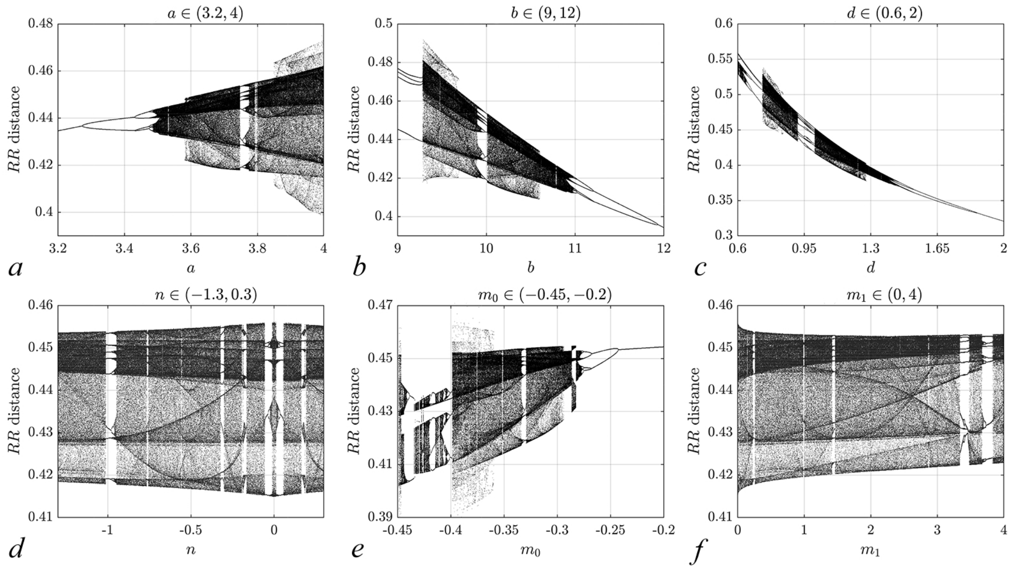

Fig. 7. Bifurcation diagrams for different parameters of the modified ECG model

parameters of the simulation are as follows: initial conditions $x_0 = (0.01, 0.1, 0, 0, 0)$, integration step $h = 10^{-4}$, base parameter set:

$$(a, b, d, n, m_0, m_1) = (3.75, 10, 1, -1, -0.33, 0.25, 5.5, 10),$$

transient rejection time $TT = 300$, computation time for analysis $CT = 150$, minimum distance values for DBSCAN $\varepsilon = 0.001$. The resulting two-dimensional diagrams are shown in Fig. 8.

The obtained diagrams show that due to the rich source dynamics of the chaotic system driving the ECG signal generator, it is possible to implement ECG signals with quite different dynamics.

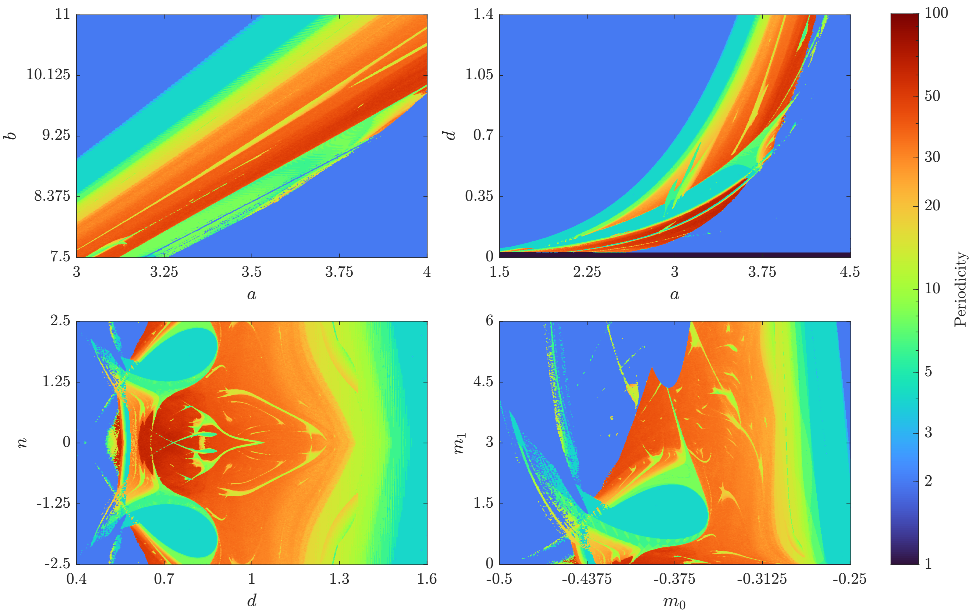

Fig. 8. Bifurcation diagrams for different parameters of the modified ECG model (color online)

Conclusions

In this study, we presented a chaotic phenomenological model of ECG based on the McSharry model. The main novelty of the work is the use of a 4th-order hyperchaotic model of a circuit with a memristor to generate a signal driving the ECG waveform instead of a harmonic oscillator. This technique allows adding natural heart rhythm variability into the model without utilizing any (pseudo-) random number generators. The test results demonstrate the effectiveness of this model in reproducing key features of real patient ECG data. The phase portrait of the proposed rhythm generator shows similarity to the phase portrait of the system reconstructed from real ECG records. Additionally, the distribution of RR intervals generated by the model accurately reflects real patient data, demonstrating a correspondence to the observed physiological rhythms and is easily tuned by selecting various model parameters.

This chaotic ECG model can be easily implemented in both digital and analog equipment by using a real memristor or its analog emulation. This dual implementation approach ensures that the model can be adapted for use in computational environments and physical devices, making it suitable for a variety of research and clinical applications. Typical applications of the proposed model may include testing ECG monitoring devices without contacting real patients, education, and creating augmented datasets for machine learning applications.

Further research will include a detailed analysis of various types of cardiac arrhythmias and the approximation of their distributions using dynamic chaos. It is also planned to implement the proposed model as a finished device and develop methods for diagnosing critical conditions and their precursors using the developed model.

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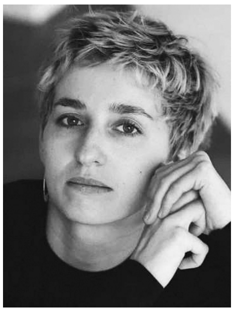
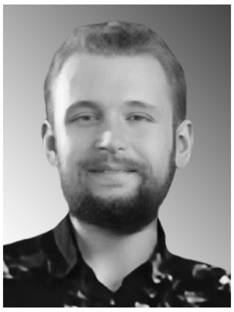


Копец Екатерина Евгеньевна — кандидат технических наук (2022, СПбГЭТУ «ЛЭТИ»). Работает на кафедре автоматизированного проектирования Санкт-Петербургского государственного электротехнического университета в должности доцента и в Молодежном научно-исследовательском институте (СПбГЭТУ «ЛЭТИ») в качестве старшего научного сотрудника. Научные интересы: приборостроение, робототехника, системы управления и идентификации данных.

Россия, 197022 Санкт-Петербург, ул. Профессора Попова, 5Ф
Санкт-Петербургский государственный электротехнический университет «ЛЭТИ»
E-mail: eekopets@etu.ru
ORCID: 0000-0002-5953-255X
AuthorID (eLibrary.Ru): 1073167

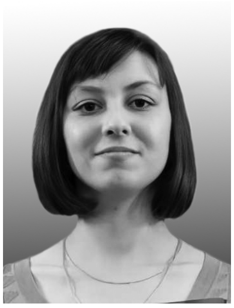
Рыбин Вячеслав Геннадьевич — кандидат технических наук (2024, СПбГЭТУ «ЛЭТИ»). Доцент кафедры автоматизированного проектирования и научный сотрудник Молодежного научно-исследовательского института Санкт-Петербургского государственного электротехнического университета «ЛЭТИ». Научные интересы: моделирование и анализ динамических систем, теория хаоса и ее приложения, хаотические системы связи, нелинейная динамика, проектирование аналоговых и цифровых схем.

Россия, 197022 Санкт-Петербург, ул. Профессора Попова, 5Ф
Санкт-Петербургский государственный электротехнический университет «ЛЭТИ»
E-mail: vgrybin@etu.ru
ORCID: 0000-0002-6515-0224
AuthorID (eLibrary.Ru): 938820



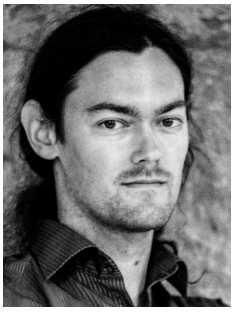
Васильченко Олег Вадимович — окончил бакалавриат факультета компьютерных технологий и информатики Санкт-Петербургского государственного электротехнического университета «ЛЭТИ» (2023), в настоящее время — магистрант СПбГЭТУ «ЛЭТИ». Научные интересы: нейронные сети, биоинспирированный анализ аудиосигналов, нелинейная динамика в сложных системах и методы метаэвристической оптимизации.

Россия, 197022 Санкт-Петербург, ул. Профессора Попова, 5Ф
Санкт-Петербургский государственный электротехнический университет «ЛЭТИ»
E-mail: ovvasilchenko@etu.ru
ORCID: 0009-0008-8295-2107



Куртова Карина Александровна — родилась в Санкт-Петербурге (2002). Окончила бакалавриат факультета компьютерных наук и технологий Санкт-Петербургского государственного электротехнического университета «ЛЭТИ» (2024). В настоящее время обучается в магистратуре СПбГЭТУ «ЛЭТИ».

Россия, 197022 Санкт-Петербург, ул. Профессора Попова, 5Ф
Санкт-Петербургский государственный электротехнический университет «ЛЭТИ»
E-mail: kakurtova@stud.etu.ru
ORCID: 0009-0001-6680-5763



Каримов Тимур Искандарович — кандидат технических наук (2018, СПбГЭТУ «ЛЭТИ»). Доцент кафедры автоматизированного проектирования и старший научный сотрудник Молодежного научно-исследовательского института Санкт-Петербургского государственного электротехнического университета «ЛЭТИ». Научные интересы: системы связи, основанные на хаосе; моделирование нелинейных систем; идентификация нелинейных систем; методы численного интегрирования; разработка датчиков, основанных на хаотических генераторах.

Россия, 197022 Санкт-Петербург, ул. Профессора Попова, 5Ф
Санкт-Петербургский государственный электротехнический университет «ЛЭТИ»
E-mail: tikarimov@etu.ru
ORCID: 0000-0002-9860-8211
AuthorID (eLibrary.Ru): 767428

Каримов Артур Искандарович — кандидат технических наук (2017, СПбГЭТУ «ЛЭТИ»), магистр английского языка (2014, СПбГЭТУ «ЛЭТИ»). Доцент кафедры автоматизированного проектирования Санкт-Петербургского государственного электротехнического университета «ЛЭТИ». Научные интересы: робототехника, искусственный интеллект, идентификация систем и теория хаоса.

Россия, 197022 Санкт-Петербург, ул. Профессора Попова, 5Ф
Санкт-Петербургский государственный электротехнический университет «ЛЭТИ»
E-mail: aikarimov@etu.ru
ORCID: 0000-0002-2591-0962
AuthorID (eLibrary.Ru): 767425

Бутусов Денис Николаевич — кандидат технических наук, доцент. Заведующий кафедрой автоматизированного проектирования и директор Молодежного научно-исследовательского института Санкт-Петербургского государственного электротехнического университета «ЛЭТИ». Входит в состав редколлегии журналов Journal of Control, Automation and Electrical Systems и Chaos Theory and Applications Journal.

Россия, 197022 Санкт-Петербург, ул. Профессора Попова, 5Ф
Санкт-Петербургский государственный электротехнический университет «ЛЭТИ»
E-mail: dnbutusov@etu.ru
ORCID: 0000-0002-8941-4220
AuthorID (eLibrary.Ru): 582555