

# On exchange-correlation energy in DFT scenarios

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Motivated by the considerable importance of material properties in modern condensed matter physics research, and using techniques of the  $N_e$ -electron systems in terms of the electron density  $n_{\sigma e}(r)$  needed to obtain the ground-state energy  $E_{e0}$  in Density Functional theory scenarios, we approach the exchange-correlation energy  $E_{xc}[n_{\sigma e}(r)]$  by considering the interelectronic position corrections  $\Delta r_x^{\uparrow\uparrow, \uparrow\downarrow} = \lambda_x |\delta r^{\uparrow\uparrow} - \delta r^{\uparrow\downarrow}|$  and  $\Delta r_c^{e_i e_j \neq i} = \lambda_c |r - r'|^{-(N_e - 1)^{-1}}$  corresponding to the spin and the Coulomb correlation effects, respectively, through the electron-electron potential energy. Exploiting such corrections, we get approximate expressions for the exchange  $E_x[n_{\sigma e}]$  and the correlation  $E_c[n_{\sigma e}]$  functional energies which could be interpreted in terms of magnetic and electric dipole potential energies associated with the charge density  $n_{\sigma e}(r)$  described by inverse-square potential behaviors. Based on these arguments, we expect that such obtained exchange-correlation functional energy could be considered in the Local Density Approximation functional as an extension to frame such interelectronic effects.

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In this work, motivated by the considerable importance of material properties in modern condensed matter physics research, and using techniques of the  $N_e$ -electron systems in terms of the electron density  $n_{\sigma e}(r)$  needed to obtain the ground-state energy  $E_{e0}$  in density functional theory scenarios [1–5], we approach the exchange-correlation energy  $E_{xc}[n_{\sigma e}(r)]$ . For that, we first present the study of the total electronic energy of many-electron  $N_e$  systems from the generalized Schrödinger equation of the many-electron wavefunction  $\Psi_{\sigma e}(x_i)$ , discuss the early attempts used to overcome the computational problem by means of the introduction of the electron density  $n_{\sigma e}(r)$  reducing the dimensionality of the system, and reconsider the Density Functional Theory (DFT) computational scheme of the  $N_e$ -electron systems using the electron density  $n_{\sigma e}(r)$  for getting the ground state energy  $E_{e0}$  [5–8]. Then, by considering the interelectronic position corrections  $\Delta r_x^{\uparrow\uparrow, \uparrow\downarrow} = \lambda_x |\delta r^{\uparrow\uparrow} - \delta r^{\uparrow\downarrow}|$  and  $\Delta r_c^{e_i e_j \neq i} = \lambda_c |r - r'|^{-(N_e - 1)^{-1}}$  corresponding to the spin and the Coulomb correlation effects, respectively, through the electron-electron potential energy. Employing such corrections, we get approximate expressions for the exchange  $E_x[n_{\sigma e}]$  and the correlation  $E_c[n_{\sigma e}]$  functional energies. By making contact with some known physical energy behaviour systems, a close inspection shows that

these obtained energies could be interpreted in terms of magnetic and electric dipole potential energies associated with the charge density  $n_{\sigma e}(r)$  described by inverse-square potential behaviors [9, 10]. Such inverse-square potential have been largely investigated in different physical scales including high energy physics and related topics [11–13]. Based on these arguments, we expect that such obtained exchange-correlation functional energy could be considered in the Local Density Approximation functional as an extension to frame such interelectronic effects [14, 15]. Particularly, we think that these dipole-like potentials should be implemented in the generalized Schrödinger equation in the DFT framework providing interelectronic interaction energy corrections as well as possible new gates to explore other physical features.

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