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SUBTERAHERTZ DIELECTRIC AND PLASMA-DIELECTRIC CHERENKOV AMPLIFIERS ON RELATIVISTIC HIGH-CURRENT ELECTRON BEAMS

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Abstract. Amplifiers with dielectric and plasma-dielectric filling, based on excitation of surface electromagnetic waves at dielectric boundary by relativistic electron beam, are considered. The focus is on the subterahertz operating frequency range. In linear approximation dispersion equation is obtained and areas of amplified frequencies and field structure are determined. Two limiting modes of Cherenkov beam instability depending on electron beam density are identified. The role of plasma near the dielectric boundary was evaluated. Efficiency of conversion of energy of directed motion of electron beam into energy of electromagnetic waves is determined on the basis of solution of shortened system of nonlinear equations. Schemes of radiation output from amplifier working area are proposed.

Keywords: *Plasma-dielectric waveguide, subterahertz radiation, relativistic electron beam, Cherenkov effect*

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1. INTRODUCTION. PROBLEM STATEMENT

Intensive theoretical and experimental studies begun in the 1970s, aimed at obtaining powerful coherent electromagnetic radiation in the microwave range using high-current electron beams propagating in electrodynamic systems with plasma filling, have been undoubtedly successful. Currently, there are operating sub-gigawatt plasma sources of electromagnetic radiation in the centimeter wavelength range [1–3]. During their creation, a new direction of applied physics emerged — high-current relativistic plasma microwave electronics [4], which continues to actively develop today [5]. Therefore, it is completely natural to desire to "advance" the existing successes and achievements in the microwave field to a higher frequency region, for example, to the subterahertz or even terahertz ranges. The study of the possibility of advancing existing Cherenkov plasma sources of electromagnetic radiation into the subterahertz range is addressed in work [6].

This paper examines Cherenkov emitters on dense relativistic electron beams, using dielectric and plasma-dielectric waveguides as electrodynamic systems. The idea of using combined plasma-dielectric structures for wave deceleration in Cherenkov emitters was considered, for example, in works [7–9]. We will mainly consider the interaction of an electron beam with high modes of dielectric waveguides when they belong to the type of so-called spatially developed (multi-wave) electrodynamic systems [10], since the wavelengths of high (and therefore high-frequency) waveguide modes are small compared to its transverse dimension.

When the inequality $c/\sqrt{\varepsilon_d} < u$, is satisfied, where u is beam velocity, ε_d is dielectric permittivity, sufficient wave deceleration exists even without plasma. This case represents the main interest in the present work. If the opposite inequality $c/\sqrt{\varepsilon_d} > u$, is satisfied, then wave deceleration is provided only by plasma, which is less interesting for us here. We consider the case of simultaneous presence of both dielectric and plasma in the waveguide, if only because in strong high-frequency fields, breakdown develops on the dielectric surface, leading to plasma formation in waveguide regions adjacent to the dielectric [11,12]. Plasma significantly changes the electrodynamics of the dielectric waveguide, which should be taken into account within the framework of the problems outlined above. Essentially, this work is a continuation of our work [6], which addressed the problem of increasing the operating frequency of emitters using only high-density plasma without any dielectric inserts. Thus, in this work, we are trying to solve the same problem of frequency increase by using a different electrodynamic system[1].

Let's consider the interaction of a straight electron beam with a wave E-type of a circular cross-section waveguide with radially inhomogeneous isotropic medium filling with dielectric permittivity of the form

$$\varepsilon_{ij}(\omega,r) = \varepsilon(\omega,r)\delta_{ij}, \qquad i,j = r,\varphi,z, \tag{1}$$

where r,φ,z are cylindrical coordinates, and $\varepsilon(\omega,r)$ is some function of the radial coordinate r (and frequency ω). Let's direct the axis $0Z$ along the waveguide axis, coinciding with the beam direction, and define the azimuthally symmetric longitudinal component of the electric field intensity by the formula

$$E_z(t,z,r) = \frac{1}{2}[E(r)\exp(-i\omega t + ik_z z) + C.C.]. \tag{2}$$

From Maxwell's equations with the dielectric permittivity tensor (1) follows the equation for the complex amplitude $E(r)$ in formula (2)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{\varepsilon(\omega,r)}{\chi^2(\omega,r)}\frac{dE}{dr}\right) - \varepsilon(\omega,r)E = 0, \tag{3}$$

where $\chi^2(\omega,r) = k_z^2 - \varepsilon(\omega,r)\omega^2/c^2$.

We will consider two variants of waveguide medium filling. In the first variant

$$\varepsilon(\omega,r) = \begin{cases} \varepsilon_d, & 0 < r < r_0, \\ \varepsilon_p = 1 - \omega_p^2/\omega^2, & r_0 < r < R, \end{cases} \tag{4}$$

where ω_p is a constant electron Langmuir frequency. According to (4), in the inner region of the waveguide $r < r_0$ there is a dielectric with permittivity $\varepsilon_d > 1$, and in the outer region — cold collisionless electron plasma. This filling variant will be called the direct geometry case. In the second filling variant, called the inverse geometry case,

$$\varepsilon(\omega,r) = \begin{cases} \varepsilon_p = 1 - \omega_p^2/\omega^2, & 0 < r < r_0, \\ \varepsilon_d, & r_0 < r < R, \end{cases} \tag{5}$$

i.e., the inner region of the waveguide is filled with plasma.

As can be seen from formulas (1), (4), and (5), the plasma in the waveguide with a dielectric insert is unmagnetized. In the above-mentioned works [7–9], the opposite case was considered — fully magnetized plasma. Below it will be shown that the cases of plasma without external magnetic field and plasma in infinitely strong external magnetic field differ significantly from each other. The matter here is in the different set of eigenwaves of plasma-dielectric waveguides.

Directly from equation (3), the following boundary conditions at the interface between dielectric and plasma are obtained:

$$\{E\}_{r=r_0} = 0, \qquad \left\{\frac{\varepsilon(\omega,r)}{\chi^2(\omega,r)}\frac{dE}{dr}\right\}_{r=r_0} = 0. \tag{6}$$

Here and further, curly brackets denote the difference of expressions on both sides of the boundary, that is, for example, $\{E\}_{r=r_0} = E(r_0+0) - E(r_0-0)$. Another boundary condition $E(R) = 0$ is written at the perfectly conducting waveguide boundary.

As can be seen from (1), the medium in the waveguide is isotropic, which in relation to plasma means it is unmagnetized. However, in any of the known Cherenkov plasma emitters, there is necessarily some external magnetic field required for obtaining and transporting high-current relativistic electron beam. Plasma unmagnetization assumes the fulfillment of inequalities [14]

$$\Omega_e^2 \ll \omega^2, \; \Omega_e\omega_p^2 \ll \omega^3, \tag{7}$$

where Ω_e is the electron cyclotron frequency. In this work, we are only interested in the region of

[1] Wave amplification in dielectric waveguides without plasma in the low-frequency range was considered in work [13].

sufficiently high frequencies, therefore we consider inequalities (7) to be unconditionally fulfilled. The situation is different with the electron beam. The densities of electron beams used in plasma emitters are small compared to plasma densities. Therefore, assuming the inequalities are fulfilled [7, 14]

$$\Omega_e^2 \gg \omega_b^2\gamma^{-3}, |\,\delta ku\,|^2 \quad (8)$$

we consider the electron beam to be fully magnetized. Moreover, following experimental conditions, we use the model of an infinitely thin tubular electron beam. Here ω_b is the Langmuir frequency of beam electrons, $\gamma = (1 - u^2 / c^2)^{-1/2}$, δ_b is the thickness of the tubular beam, and is δk is the spatial increment (gain coefficient) of resonant Cherenkov beam instability.

The current density of an infinitely thin tubular magnetized electron beam is defined by the formula

$$j_z(t,z,r) = \delta_b \delta(r - r_b) j_b(t,z;r_b), \quad (9)$$

where $j_b(t,z;r_b)$ — is a function that requires beam dynamics equations to find, and the parametric dependence on the tubular beam radius r_b indicates that the field (2) acts on the electrons of the thin beam precisely at point $r = r_b$. The presence of current with density (9) in the waveguide leads to the following jump in the azimuthal component of the magnetic field induction

$$\left\{B_\varphi(t,z,r)\right\}_{r=r_b} = \frac{4\pi}{c}\delta_b j_b(t,z;r_b). \quad (10)$$

The azimuthal component of the magnetic field induction is determined by a formula of type (2) with complex amplitude

$$B(r) = -i\varepsilon(\omega,r)\frac{\omega}{c\chi^2(\omega,r)}\frac{dE}{dr}. \quad (11)$$

Substituting the formula of type (2) with complex amplitude (11) into (10) and assuming that the electron beam passes through one of the plasma regions of the waveguide, we find the following boundary conditions for function $E(r)$ on the electron beam:

$$\left\{E\right\}_{r=r_b} = 0, \quad \left\{\frac{dE}{dr}\right\}_{r=r_b} = \delta_b \frac{\chi_p^2}{\varepsilon_p}\frac{4\pi i}{\omega}\left\langle j_b(r_b)\right\rangle, \quad (12)$$

where

$$\left\langle j_b(r_b)\right\rangle = \frac{\omega}{\pi}\int\limits_0^{2\pi/\omega} j_b(t,z;r_b)\exp\left(i\omega t - ik_z z\right)dt \quad (13)$$

is the space-time Fourier harmonic of the beam current density, and $\chi_p^2 = k_z^2 - \varepsilon_p\omega^2 / c^2$. In (12), an obvious condition for the continuity of the function itself on the electron beam is added $E(r)$. In the linear approximation, to calculate function (13), one can use the known expression for electron beam conductivity obtained in the hydrodynamic model [7, 14], which gives

$$\frac{4\pi i}{\omega}\left\langle j_b(r_b)\right\rangle = -\frac{\omega_b^2\gamma^{-3}}{(\omega - k_z u)^2}E(r_b). \quad (14)$$

The procedure for calculating function (14) in nonlinear theory will be described below.

2. DISPERSION EQUATION OF LINEAR THEORY FOR THE CASE OF DIRECT GEOMETRY

Let us now proceed to consider the waveguide in the case of direct geometry (4). Since the beam passes through the plasma region of the waveguide, we assume that $r_0 < r_b < R$. Taking into account the boundary conditions $r = r_0$ and $r = R$, the solution of equation (3) in different regions of the waveguide can be written as

$$E(r) = \begin{cases} AI_0(\chi_d r), & r < r_0, \\ A\chi_p r_0[UI_0(\chi_p r) + VK_0(\chi_p r)], & r_0 < r < r_b, \\ BF_0(\chi_p r), & r_b < r < R, \end{cases} \quad (15)$$

where

$$\begin{gathered} V(\omega,k_z) = I_0(\chi_d r_0)I_1(\chi_p r_0) - \\ -\frac{\varepsilon_d\chi_p}{\varepsilon_p\chi_d}I_1(\chi_d r_0)I_0(\chi_p r_0), \\ U(\omega,k_z) = I_0(\chi_d r_0)K_1(\chi_p r_0) + \\ +\frac{\varepsilon_d\chi_p}{\varepsilon_p\chi_d}I_1(\chi_d r_0)K_0(\chi_p r_0), \\ F_0(\chi_p r) = K_0(\chi_p r) - I_0(\chi_p r)\frac{K_0(\chi_p R)}{I_0(\chi_p R)}, \end{gathered} \quad (16)$$

A and B constants, $I_0(x)$ and $K_0(x)$ are Infeld and MacDonald functions, and $\chi_d^2 = k_z^2 - \varepsilon_d\omega^2/c^2$. Substituting solutions (15) into the boundary conditions (12), we obtain the following relations:

$$A = B\frac{F_0(\chi_p r_b)}{\chi_p r_0 I_0(\chi_p r_b) D_0(\omega, k_z; r_b)}, \quad (17)$$

$$D_0(\omega, k_z; R)B =$$
$$= -D_0(\omega, k_z; r_b) I_0(\chi_p r_b)\delta_b r_b \frac{\chi_p^2}{\varepsilon_p}\frac{4\pi i}{\omega}\langle j_b(r_b)\rangle, \quad (18)$$

where

$$D_0(\omega, k_z; x) = U(\omega, k_z) + V(\omega, k_z)K_0(\chi_p x)/I_0(\chi_p x). \quad (19)$$

According to formula (15), relation (17) determines the field structure of the wave excited by the electron beam in the plasma-dielectric waveguide. The main one is certainly relation (18), which can be interpreted as the excitation equation of the waveguide by the beam (see below).

It is easy to see that equation

$$D_0(\omega, k_z; R) = 0 \quad (20)$$

is the dispersion equation determining the frequencies of the natural waves of the plasma-dielectric waveguide without the beam. Indeed, in the absence of a beam (for example, at $\delta_b = 0$) equation (18) has a non-trivial solution $B \neq 0$ only at $D_0(\omega, k_z; R) = 0$.

In the linear approximation, using formulas (14) and (15), from (18) we obtain the following dispersion equation for determining complex spectra of a plasma-dielectric waveguide with a thin tubular straight electron beam in the case of direct geometry:

$$D_0(\omega, k_z; R) - \frac{\delta_b r_b}{R^2}W_0(\omega, k_z)\frac{\omega_b^2\gamma^{-3}}{(\omega - k_z u)^2} = 0, \quad (21)$$

where

$$W_0(\omega, k_z) = R^2\frac{\chi_p^2}{\varepsilon_p}D_0(\omega, k_z; r_b)I_0(\chi_p r_b)F_0(\chi_p r_b). \quad (22)$$

If the wavelengths excited by the electron beam are small compared to the waveguide radius ($c/\omega \ll R$), which is exactly the case for high modes, then the dispersion equation (20) significantly simplifies. Indeed, in (19), we obtain the following dispersion equation:

$$D_0(\omega, k_z; \infty) = U(\omega, k_z) = 0. \quad (23)$$

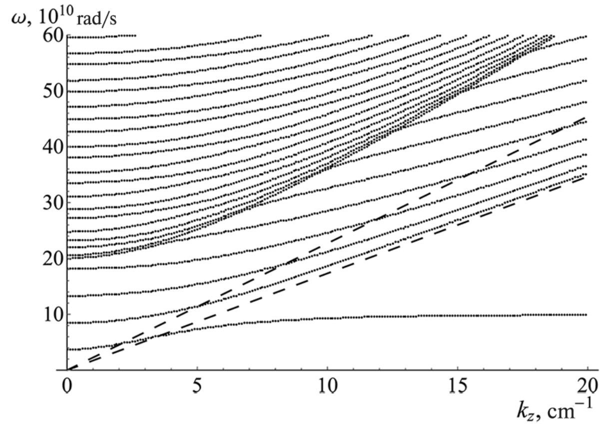

Fig. 1. Characteristic dispersion curves of the plasma-dielectric waveguide and lines (lower dashed line) $\omega = k_z c/\sqrt{\varepsilon_d}$ and (upper dashed line) $\omega = k_z u$

3. WAVE AMPLIFICATION IN A PLASMA-DIELECTRIC WAVEGUIDE IN THE REGIME OF SINGLE-PARTICLE CHERENKOV EFFECT. CASE OF DIRECT GEOMETRY

Let us investigate spatial wave amplification in a plasma-dielectric waveguide with a beam in the case of direct geometry, for which we will solve equation (21) with respect to the complex wave number $k_z(\omega)$ at real frequency ω. Assuming, as is usually done in Cherenkov interaction of beams with waves of any nature [7]

$$k_z = k_{z\omega} + \delta k, \qquad |\delta k| \ll k_{z\omega}, \quad (24)$$

where $k_{z\omega} = \omega/u$, we transform equation (21) into a cubic equation for the complex amplification coefficient δk

$$\left(D_0(\omega, k_{z\omega}; R) + \frac{\partial D_0(\omega, k_{z\omega}; R)}{\partial k_{z\omega}}\delta k\right)\delta k^2 =$$
$$= \frac{\delta_b r_b}{R^2}\left(W_0(\omega, k_{z\omega}) + \frac{\partial W_0(\omega, k_{z\omega})}{\partial k_{z\omega}}\delta k\right)\frac{\omega_b^2\gamma^{-3}}{u^2}. \quad (25)$$

Let the condition of single-particle Cherenkov resonance be satisfied (i.e., the wave is in Cherenkov resonance with the electron), and therefore

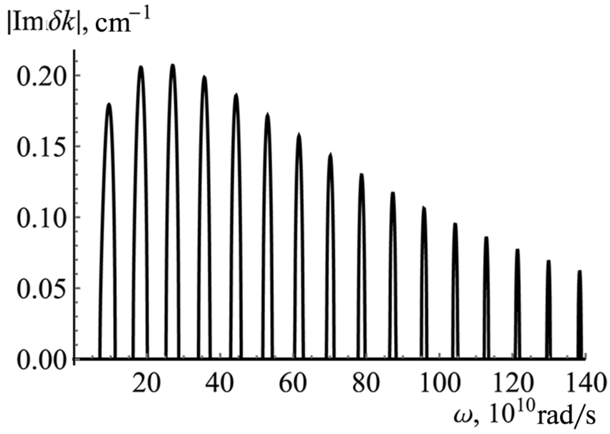

Fig. 2. Gain coefficient dependence on frequency in a dielectric waveguide without plasma

$$D_0(\omega, k_{z\omega}; R) = 0, \qquad (26)$$

and the beam density is so low that the second term in brackets on the right side of the dispersion equation (25) can be neglected. Then, for the complex amplification coefficient at the frequency of single-particle Cherenkov resonance, we obtain the following expression:

$$\delta k(\omega_0) = \frac{1 - i\sqrt{3}}{2}\frac{\omega_0}{u}\left|\frac{\delta_b r_b}{R^2}\frac{\omega_b^2\gamma^{-3}}{\omega_0^2} \times W_0(\omega_0, k_0)\left(k_0\frac{\partial D_0(\omega_0, k_0; R)}{\partial k_0}\right)^{-1}\right|^{1/3}, \qquad (27)$$

where ω_0 is the solution of equation (26), and $k_0 = \omega_0 / u$. The amplification coefficient (27) has the structure typical for any single-particle Cherenkov effect. The condition for applicability of solution (27), i.e., the condition for single-particle amplification is the inequality

$$\left|\frac{1}{W_0(\omega_0, k_0)}\frac{\partial W_0(\omega_0, k_0)}{\partial k_0}\delta k(\omega_0)\right| n1. \qquad (28)$$

To determine the resonant frequencies ω_0 one should solve equation (26) with respect to frequency *w*. The frequencies ω_0 are most clearly represented on the dispersion diagram (Fig. 1), which shows the dispersion curves — solutions of the dispersion equation (20), the line $\omega = k_z c / \sqrt{\varepsilon_d}$ (lower dashed line) and the line $\omega = k_z u$ (upper dashed line). Frequencies ω_0 are given by the intersection points of dispersion curves with the line $\omega = k_z u$. The calculation was performed for a system with the following

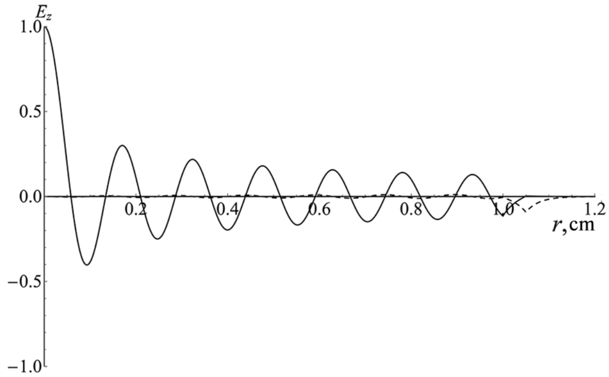

Fig. 3. Longitudinal component of the electric field mode $E_{0.13}$ during its resonant excitation by electron beam: ReEz solid line, ImEz dashed line

parameters: $\omega_p = 2\cdot10^{11}$ rad/s, $r_0 = 1$ cm, $R = 3$ cm, $\varepsilon_d = 3$, $u = 2.27 \cdot 10^{10}$ cm/s. The figure shows resonances only with the six lowest modes (although resonance exists at arbitrarily high modes), the first of which is plasma mode.

Let's pay attention to the lowest dispersion curve in Fig. 1 — it corresponds to the surface plasma wave[2]. As we can see, the cutoff frequency of this wave is not zero, which is characteristic only for a waveguide with plasma filling in the external region [15], which is exactly the case in direct geometry. Therefore, the Cherenkov resonance of the electron beam with such a surface plasma wave always exists, which must be taken into account when developing Cherenkov dielectric amplifiers with an internal dielectric insert. Indeed, with high power of the amplified signal, the plasma arising from breakdown fundamentally changes the electrodynamic properties of the system — instability appears on the low-frequency surface plasma wave, which can suppress amplification in the high-frequency region.

4. REGIME OF COLLECTIVE CHERENKOV EFFECT

In principle, equation (21) also describes the collective Cherenkov effect [7,16]. To understand this issue, equation (21) should be written in such a way as to explicitly highlight the dispersion function of the waveguide without beam (it has already been highlighted) and the dispersion function of the beam's Langmuir waves, which can be done easily

[2] The term "surface wave" refers only to the frequency range $\omega < k_z c$.

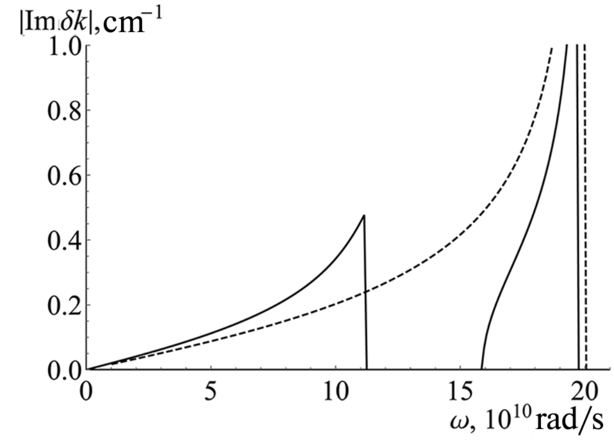

Fig. 4. Gain coefficients versus frequency: homogeneous plasma filling – dashed line; plasma-dielectric filling – solid line

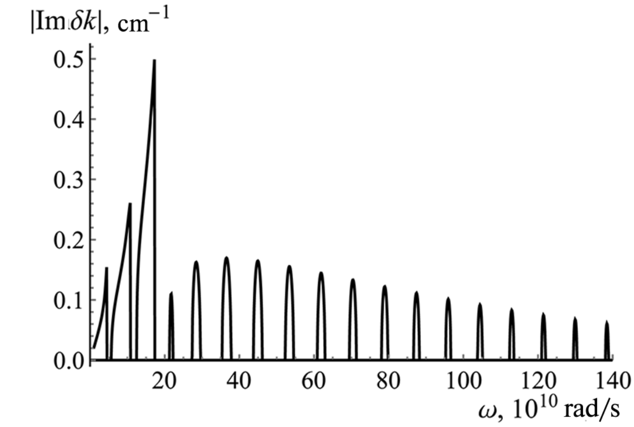

Fig. 5. Dependence of gain coefficient on frequency in plasma-dielectric waveguide.

using expressions (19) and (22). As a result, equation (21) transforms to the form

$$D_0(\omega,k_z;R)D_b(\omega,k_z) = \delta_b r_b \frac{\chi_p^2}{\varepsilon_p}\omega_b^2\gamma^{-3}\theta(\omega,k_z), \quad (29)$$

where

$$D_b(\omega,k_z) = (\omega - k_z u)^2 - \Omega_b^2(\omega,k_z)/\varepsilon_p \quad (30)$$

is the aforementioned dispersion function of the beam's Langmuir waves, where

$$\begin{aligned} \Omega_b^2(\omega,k_z) &= \delta_b r_b \chi_p^2 \omega_b^2 \gamma^{-3} I_0^2(\chi_p r_b) \times \\ &\times \left(\frac{K_0(\chi_p r_b)}{I_0(\chi_p r_b)} - \frac{K_0(\chi_p R)}{I_0(\chi_p R)}\right), \\ \theta(\omega,k_z) &= V(\omega,k_z) I_0^2(\chi_p r_b) \times \\ &\times \left(\frac{K_0(\chi_p r_b)}{I_0(\chi_p r_b)} - \frac{K_0(\chi_p R)}{I_0(\chi_p R)}\right)^2. \end{aligned} \quad (31)$$

The dispersion equation determining the frequencies of Langmuir waves (charge density waves) of a thin tubular magnetized electron beam in a plasma waveguide without a dielectric cylinder is $D_b(\omega,k_z) = 0$. The left side of the dispersion equation (29) is written in the "canonical" form of coupled waves equation — waves of the dielectric cylinder in plasma and waves of the tubular beam in plasma. The wave coupling coefficient is contained in the value $\theta(\omega,k_z)$.

When the inequality $\Omega_b^2 \ll \omega^2 \sim |k_z u|^2$ is satisfied, the spectrum of the slow charge density wave of the beam can be determined by the formula[3]

$$k_z^b(\omega) = k_{z\omega}\left(1 + \frac{\Omega_b(\omega,k_{z\omega})}{\omega\sqrt{\varepsilon_p}}\right). \quad (32)$$

The resonance frequency ω_0^b of one of the waves of the plasma-dielectric waveguide and the slow beam wave is determined from the equation $D_0\left(\omega,k_z^b(\omega);R\right) = 0$. At the resonance frequency, the solution of equation (29) is sought in the form

$$k_z(\omega_0^b) = k_0^b + \delta k, \quad (33)$$

where $k_0^b = k_z^b(\omega_0^b)$. Substituting (33) into equation (29), we find the following expression for the resonant gain coefficient in the regime of collective Cherenkov effect:

$$\delta k(\omega_0^b) = -i\left|\frac{1}{2}\delta_b r_b \frac{\chi_p^2}{\varepsilon_p}\frac{\omega_b^2\gamma^{-3}}{u\Omega_b(\omega_0^b,k_0^b)} \times \theta(\omega_0^b,k_0^b)\left(\frac{\partial U(\omega_0^b,k_0^b)}{\partial k_0^b}\right)^{-1}\right|^{1/2}. \quad (34)$$

The condition for the applicability of solution (34), i.e., the condition for collective amplification, is the inequality[4]

$$|\,\delta k(\omega_0^b)u\,| = \Omega_b(\omega_0^b,k_0^b). \quad (35)$$

[4] We do not explicitly present inequalities (28) and (35) here due to their complexity. The fulfillment of these inequalities was verified by us during numerical calculations. At those densities and radii of the electron beam that were taken during numerical calculations, the weak inequality (28) was usually satisfied.

5. WAVE GAIN COEFFICIENTS BY ELECTRON BEAM FOR THE CASE OF DIRECT GEOMETRY OF PLASMA-DIELECTRIC WAVEGUIDE

Let us now consider the results of calculating wave gain coefficients in a waveguide with the following parameters: waveguide radius $R = 3$ cm, dielectric cylinder radius $r_0 = 1$ cm, dielectric permittivity $\varepsilon_d = 3$. Let's take the following beam parameters: velocity $u = 2.27 \cdot 10^{10}$ cm/s, Langmuir frequency $\omega_b = 2.5 \cdot 10^{10}$ rad/s, average radius $r_b = 1.05$ cm, thickness $\delta_b = 0.1$ cm (the current of such beam is about 1 kA). For the case when there is no plasma in the waveguide, the frequency dependence of the gain coefficient is shown in Fig. 2.

A large number of gain zones can be seen, each corresponding to Cherenkov interaction of the beam with one of the waveguide modes — from mode E_{01} to mode $E_{0,16}$. Higher gain zones, which, if not accounting for frequency dispersion of permittivity ε_d, are infinitely many, are located in the higher frequency region. The gain zones are quite narrow, although even at frequency: 10^{12} rad/s, their width is several units at 10^{10} rad/s. Moreover, the gain coefficients are quite large. Thus, at frequency: 10^{12} rad/s, the gain coefficient is about 0.1 cm^{-1}, which provides power amplification by 1000 times with an amplifier length of 35 cm.

It should be noted that the results shown in Fig. 2 were obtained for the case when the inner boundary of the electron beam $r_b - \delta_b / 2$ coincides with the dielectric boundary r_0. When the beam moves away from the dielectric boundary, due to strong wave field attenuation in the vacuum region, the gain coefficient drops sharply. This circumstance constitutes one of the main difficulties for using already implemented Cherenkov microwave emitters in higher frequency ranges. Fig. 3 shows the real and imaginary parts of the longitudinal component of the electric field mode $E_{0,13}$ at the point of maximum gain coefficient ($\omega = 112.85 \cdot 10^{10}$ rad/s, $k_z = (50.46 - 0.08i)$ cm^{-1}). It is evident that there is virtually no field in the vacuum region of the waveguide, and at $r = r_b$ the field is not large.

Let us now turn to the case when plasma is present in the waveguide region $r \in (r_0, R)$ The presence of plasma, as noted above, significantly changes the electrodynamics of the waveguide and considerably complicates the overall picture of wave amplification. To better understand this, let's consider special cases of independent interest. Let's assume that in the waveguide region $r \in (0, r_0)$ instead of a dielectric, there is the same plasma as in $r \in (r_0, R)$. The dispersion equation for this case is obtained by replacing ε_d with ε_p. It can be shown that this equation reduces to $D_b(\omega, k_z) = 0$ (see formula (30)), which is convenient to rewrite as follows:

$$1 - \frac{\omega_p^2}{\omega^2} - \frac{\Omega_b^2(\omega, k_z)}{(\omega - k_z u)^2} = 0. \tag{36}$$

If we replace $\Omega_b^2(\omega, k_z)$ with $\omega_b^2 \gamma^{-3}$, we obtain the dispersion equation describing the interaction of an unbounded electron beam with an unbounded isotropic plasma [14]. From it, for the complex wave number we have

$$k_z = \frac{\omega}{u} - i\frac{\omega}{u}\sqrt{\frac{\omega_b^2 \gamma^{-3}}{\omega_p^2 - \omega^2}}. \tag{37}$$

At $\omega < \omega_p$ formula (37) describes the spatial amplification of the longitudinal field during beam self-modulation in a medium with negative dielectric permittivity [17]. At $\omega = \omega_p$ the amplification coefficient (37) becomes infinite, which is due to the zero group velocity of the Langmuir wave in cold plasma (oscillation accumulation effect). Similar processes occur in the waveguide as well. Indeed, at $\omega < \omega_p$ the dielectric permittivity of the plasma is negative, and the longitudinal Langmuir wave with frequency $\omega(k_z) = \omega_p$ exists in the waveguide with homogeneous isotropic plasma. Figure 4 shows the modulus of the imaginary part of the wave number obtained by numerical solution of equation (36) at $\omega_p = 20 \cdot 10^{10}$ rad/s (dashed line).

Now let the plasma be present only in the region $r \in (r_0, R)$, while the rest of the waveguide is filled with a dielectric with permittivity $\varepsilon_d < c^2 / u^2$ (one can even set $\varepsilon_d = 1$). In this case, Cherenkov resonance of the beam with electromagnetic waveguide modes is impossible. However, in addition to the potential Langmuir wave $\omega = \omega_p$ due to the presence of plasma boundary $r = r_0$ a non-potential surface wave appears in the waveguide. In the shortwavelength limit, the surface wave frequency approaches $\omega_p / \sqrt{\varepsilon_d + 1}$,

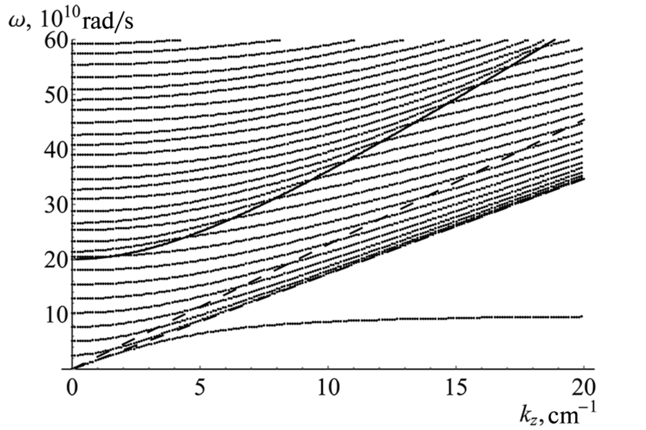

Fig. 6. Characteristic dispersion curves of the plasma-dielectric waveguide in the case of inverse geometry and lines $\omega = k_z c / \sqrt{\varepsilon_d}$ (lower dashed line) and $\omega = k_z u$ (upper dashed line)

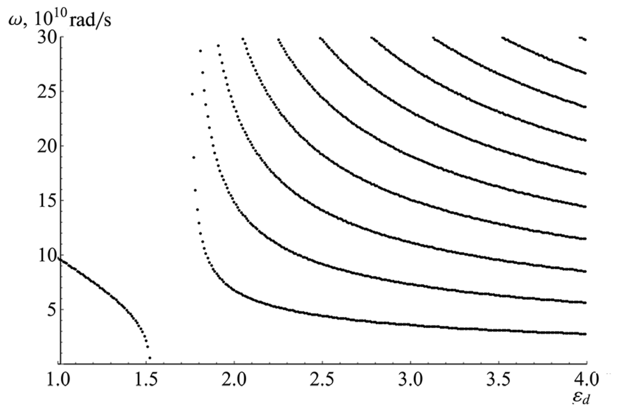

Fig. 7. Cherenkov resonances depending on the dielectric permittivity

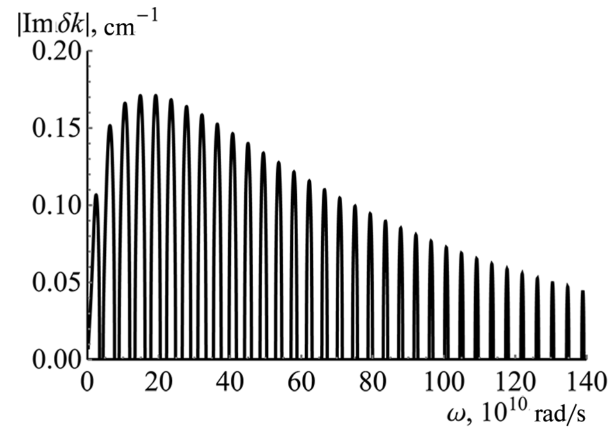

Fig. 8. Dependence of gain coefficient on frequency in a dielectric waveguide without plasma, inverse geometry

and at $k_z = 0$ there is a certain non-zero cutoff frequency (see the lower dispersion curve in Fig. 1). Therefore, there are two Cherenkov resonances —

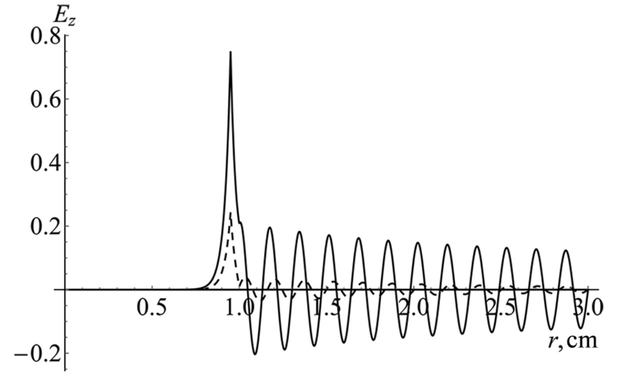

Fig. 9. Longitudinal component of electric field strength of mode $E_{0,24}$ during its resonant excitation by electron beam in inverse geometry: $\Re E_z$ – solid line, $\Im E_z$ – dashed line $\omega = 100.8 \times 10^{10}$ rad/s, $k_z = (44.9 - 0.07i)$ cm^{-1}

with bulk Langmuir and surface Langmuir waves. Consequently, two frequency regions of amplification appear. The gain coefficient for this case is shown by solid lines in Fig. 4.

Now let, as in the case of Fig. 2, $\varepsilon_d = 3$, but there is also plasma with Langmuir frequency $20 \cdot 10^{10}$ rad/s. In this case, resonances with surface plasma wave, bulk Langmuir wave, and electromagnetic modes become possible. The gain coefficient for this case is shown in Fig. 5, which is essentially a combination of Fig. 2 and 4.

One might assume that the presence of plasma in a dielectric waveguide with dielectric in region $r < r_0$ is a negative factor for solving the problem of increasing the frequency of amplified waves. Indeed, in the presence of plasma in the frequency region below the plasma frequency, the wave gain coefficients are high. The lower the wave frequency, the worse it radiates through the amplifier output boundary. Therefore, parasitic self-excitation of the amplifier in the low-frequency region is possible, which leads to beam "deterioration" and reduction in Cherenkov radiation efficiency in the high-frequency region[5]. One can, of course, take another approach: use such dense plasma that Cherenkov radiation of the surface plasma wave would occur in the sub-terahertz region. Then there would be no need for a dielectric insert in the waveguide. This approach has both advantages and disadvantages [6].

[5] As for the amplification coefficients in the high-frequency region, as can be seen from Fig. 2 and Fig. 3, plasma practically does not affect them.

6. THE CASE OF INVERSE GEOMETRY OF THE PLASMA-DIELECTRIC WAVEGUIDE

Let us now consider the inverse geometry of system (5). Assuming $r_b < r_0$ and taking into account the boundary conditions $r = r_0$ and $r = R$, the solution of equation (3) in different regions of the waveguide can be written as

$$E(r) = \begin{cases} AI_0(\chi_p r), & r < r_b, \\ B\chi_p r_0[UI_0(\chi_p r) + VK_0(\chi_p r)], & r_b < r < r_0, \\ BF_0(\chi_d r), & r_0 < r < R, \end{cases} \tag{38}$$

where

$$\begin{gathered} F_0(\chi_d r) = K_0(\chi_d r) - I_0(\chi_d r)\frac{K_0(\chi_d R)}{I_0(\chi_d R)}, \\ F_1(\chi_d r) = K_1(\chi_d r) + I_1(\chi_d r)\frac{K_0(\chi_d R)}{I_0(\chi_d R)}, \\ U(\omega,k_z) = F_0(\chi_d r_0)K_1(\chi_p r_0) - \\ -\frac{\varepsilon_d\chi_p}{\varepsilon_p\chi_d}F_1(\chi_d r_0)K_0(\chi_p r_0), \\ V(\omega,k_z) = F_0(\chi_d r_0)I_1(\chi_p r_0) + \\ +\frac{\varepsilon_d\chi_p}{\varepsilon_p\chi_d}F_1(\chi_d r_0)I_0(\chi_p r_0). \end{gathered} \tag{39}$$

Substituting solutions (38) into boundary conditions (12), we obtain the following relations:

$$B = A\times$$

$$\times\frac{I_0(\chi_p r_b)}{\chi_p r_0[U(\omega,k_z)I_0(\chi_p r_b) + V(\omega,k_z)K_0(\chi_p r_b)]}, \tag{40}$$

$$\begin{gathered} D_0(\omega,k_z)A = -[U(\omega,k_z)I_0(\chi_p r_b) + \\ +V(\omega,k_z)K_0(\chi_p r_b)]\times \\ \times\delta_b r_b\frac{\chi_p^2}{\varepsilon_p}\frac{4\pi i}{\omega}\langle j_b(r_b)\rangle, \end{gathered} \tag{41}$$

where

$$D_0(\omega,k_z) = V(\omega,k_z) \tag{42}$$

is the dispersion equation determining the frequencies of natural waves of the plasma-dielectric waveguide without beam in the case of inverse geometry.

In the linear approximation, using formulas (14) and (38), from (42) we obtain the following dispersion equation for determining the complex spectra of a plasma-dielectric waveguide with a thin tubular straight electron beam:

$$D_0(\omega,k_z) - \frac{\delta_b r_b}{R^2}W(\omega,k_z)\frac{\omega_b^2\gamma^{-3}}{(\omega - k_z u)^2} = 0, \tag{43}$$

where

$$\begin{gathered} W(\omega,k_z) = R^2\frac{\chi_p^2}{\varepsilon_p}\times \\ \times[U(\omega,k_z)I_0(\chi_p r_b) + V(\omega,k_z)K_0(\chi_p r_b)]\times \\ times I_0(\chi_p r_b). \end{gathered} \tag{44}$$

Equation (43) does not differ in form from the dispersion equation (21), therefore we omit its approximate analytical consideration here, and we will consider numerical solutions a little later.

An essential feature of inverse geometry is associated with the surface plasma wave. In the case of plasma localized in the inner region of the waveguide $r < r_0$ the cutoff frequency of the surface plasma wave equals zero. Using formulas (39) and (42), it is easy to show that in the long-wavelength limit, the frequency of the surface plasma wave is determined by the formula

$$\omega = \frac{k_z c}{\sqrt{\varepsilon_d}}\left(1 + \frac{I_1(\omega_p r_0 / c)}{I_0(\omega_p r_0 / c)}\frac{\omega_p r_0}{c}\ln\frac{R}{r_0}\right)^{-1/2}. \tag{45}$$

Considering that in the long-wavelength limit the phase velocity of the surface plasma wave is maximal, from (45) we see that at $u > c / \sqrt{\varepsilon_d}$ the surface plasma wave cannot be excited. In principle, it can be excited, but only when the inverse inequality is satisfied $u < c / \sqrt{\varepsilon_d}$, when Cherenkov resonance of the beam and electromagnetic waves is absent. For this work, the case $u < c / \sqrt{\varepsilon_d}$, i.e., weak deceleration of electromagnetic waves, is of no interest (unlike work [6], in which there is no deceleration of electromagnetic waves at all).

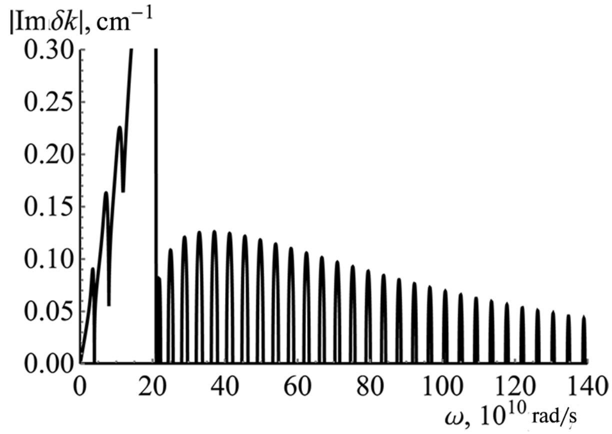

Fig. 10. Dependence of gain coefficient on frequency in plasma-dielectric waveguide, inverse geometry

The surface plasma wave in this case is not excited since because $\varepsilon_d = 3 > c^2 / u^2 \approx 1.73$. Resonances possible at different dielectric permittivities, for electron beam with velocity $u = 2.27 \cdot 10^{10}$ cm/s are shown in Fig.7.

Let's now consider wave amplification in a system with inverse filling[6]. Let the parameters of plasma-dielectric waveguide and electron beam be the same as in the case discussed in Fig. 2–5, only interchange plasma and dielectric, and set beam radius to $r_b = 0.95$ cm. The calculation results are presented in Fig. 8,9 and 10, which are analogous to Fig. 2, 3 and 5 respectively.

Notable is the similarity between Fig. 5 and 10. It was previously stated that when $u > c / \sqrt{\varepsilon_d}$, which is the case in Fig. 5 and 10, excitation of surface plasma wave is impossible in a system with inverse geometry[7]. However, resonant excitation of bulk Langmuir wave occurs for any geometry. This resonance appears as the highest maximum both in Fig. 5 and Fig. 10. Regarding the maxima in the region $\omega < \omega_p$, in case of Fig. 5 one of them is related to resonance on surface plasma wave, while in Fig. 10 all maxima are caused by resonances with electromagnetic waves (see Fig. 6). The main features of inverse geometry are more frequent placement of amplification zones and smaller width of each zone. The latter, in our opinion, is a serious disadvantage of the system with inverse geometry.

[6] Equation (42) has a degenerate solution χ_p=0, shown in Fig. 6 by a solid line. As can be seen from formulas (38), the solution χ_p=0 corresponds to zero electromagnetic field.

[7] Mathematically, this is described by the multiplier $\varepsilon^{-1}{}_p$ in expressions (22) and (44) and is related to the fact that in any geometry, the beam passes through the plasma volume.

7. NONLINEAR THEORY OF CHERENKOV WAVE AMPLIFICATION IN PLASMA-DIELECTRIC WAVEGUIDE IN CASE OF DIRECT GEOMETRY

Let us now turn to the nonlinear theory. We will limit ourselves to the case of geometry with internal localization of the dielectric insert. The nonlinear theory of Cherenkov plasma-dielectric amplifiers can be based on equation (18), which, taking into account (15), can be written as

$$D_0(\omega,k_z;R)E(r_b) = -D_0(\omega,k_z;r_b)I_0(\chi_p r_b)\times$$
$$\times F_0(\chi_p r_b)\delta_b r_b \frac{\chi_p^2}{\varepsilon_p}\frac{4\pi i}{\omega}\langle j_b(r_b)\rangle. \quad (46)$$

Here $E(r_b)$ is the field amplitude (2) at the electron beam passage location. If the field were represented not in the form of (2), but in the form

$$E_z(t,z,r) = \frac{1}{2}[E(z,r)\exp(-i\omega t + ik_{z\omega}z) + \text{C.C.}], \quad (47)$$

then in the linear approximation it would be $E(z,r) = E(r)\exp(i\delta kz)$, and in the dispersion equation, the wave number would be replaced by $k_{z\omega} + \delta k$ (see (24)). In the nonlinear case, formula (14) for the beam current density is not valid, but equation (46) is still relevant if rewritten in the following operator form:

$$D_0(\omega,\hat{k}_z;R)E(z,r_b) =$$
$$= -W_0(\omega,\hat{k}_z;r_b)\frac{\delta_b r_b}{R^2}\frac{4\pi i}{\omega}\langle j_b(z;r_b)\rangle, \quad (48)$$

where the wave number k_z is replaced by the wave number operator $\hat{k}_z = k_{z\omega} - id / dz$.

To calculate the nonlinear beam current density, we use the method of integration over initial data in the boundary value problem formulation [18]. Omitting the standard derivation procedure, which can be found, for example, in [6], we present only the final formula for the beam current density

$$\left\langle j_b(z;r_b)\right\rangle = en_{0b}u\frac{\omega}{\pi}\int_0^{2\pi/\omega}\exp\left(i\omega\tau(z,\tau_0)\right)dt_0, \quad (49)$$

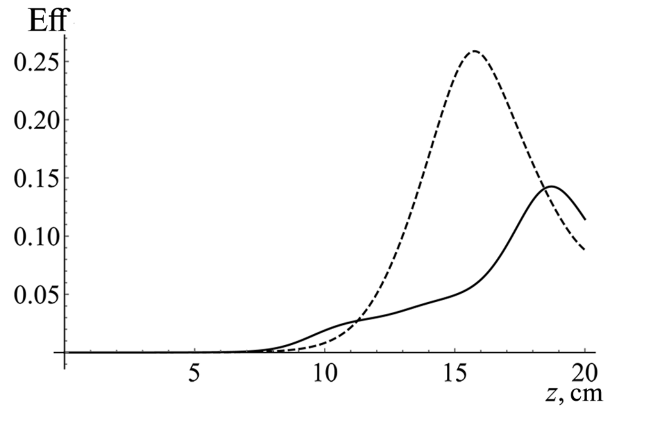

Fig. 11. Amplification efficiencies of high-frequency electromagnetic wave at frequency $\omega = 50.6 \cdot 10^{10}$ rad/s (dashed line), $\omega = 100.9 \cdot 10^{10}$ rad/s (solid line)

where n_{0b} is the unperturbed electron beam density. Function $\tau(z,\tau_0)$ is the local time at which the beam electron, which entered the waveguide cross-section $z = 0$ at moment τ_0, reaches cross-section z. This function is determined from the following equations of motion:

$$\frac{d\tau}{dz} = \frac{V}{u^2},$$
$$\frac{dV}{dz} = -\frac{e}{mu\gamma^3}\left(1 + 2\frac{u^2}{c^2}\gamma^2\frac{V}{u}\right)^{3/2} \times \tag{50}$$
$$\times\frac{1}{2}[E(z,r_b)\exp\left(-i\omega\tau(z,\tau_0)\right) + c.c.],$$

and $V = V(z,\tau_0)$ — is the perturbation of the electron beam velocity. The boundary conditions for equations (50) are

$$\tau(z = 0) = \tau_0, \qquad V(z = 0) = 0. \tag{51}$$

In addition to the electron beam entry conditions (51), the system with the wave excitation condition at the amplifier input

$$E(z = 0, r_b) = E_0. \tag{52}$$

If the electron beam was pre-modulated before injection into the waveguide, then conditions

(50) should be set as

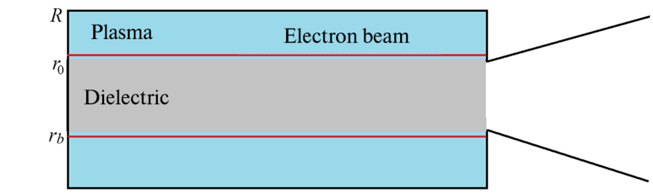

Fig. 12. Diagram of amplifier with plasma-dielectric waveguide in direct geometry

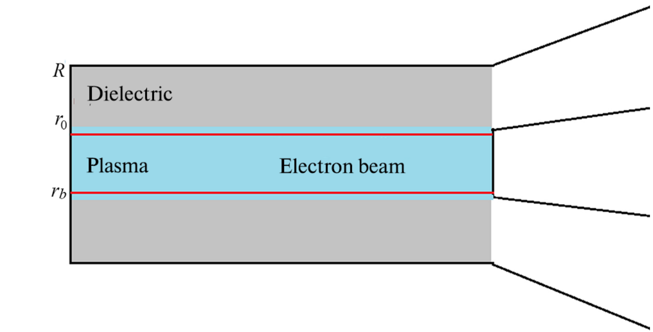

Fig. 13. Diagram of amplifier with plasma-dielectric waveguide in inverse geometry

$$\begin{gathered}\tau(z = 0) = \tau_0 + \alpha\sin(\tau_0 + p_0), \\ V(z = 0) = \beta\sin(\tau_0 + q_0),\end{gathered} \tag{53}$$

where α, β, p_0, q_0 sets the velocity modulation depth.

The pseudodifferential equation (48) is not suitable for computer modeling purposes; it should be properly simplified. Due to inequality (24), the dispersion operator $D_0(\omega, \hat{k}_z; R)$ and the right-hand side of equation (48) can be expanded in id/dz, which leads to the following equation::

$$\left(D_0 - i\frac{\partial D_0}{\partial k_{z\omega}}\frac{d}{dz}\right)E(z,r_b) =$$
$$= -\frac{\delta_b r_b}{R^2}\frac{4\pi i}{\omega}\left(W_0 - i\frac{\partial W_0}{\partial k_{z\omega}}\frac{d}{dz}\right)\langle j_b(z;r_b)\rangle, \tag{54}$$

where D_0 — function (19) at $k_z = k_{z\omega}$ and $x = R$, and W_0 — function (22) at $k_z = k_{z\omega}$. Equation (54) and equations (50) form a closed nonlinear system of equations for the Cherenkov plasma-dielectric amplifier. The efficiency of electromagnetic wave amplification by electron beam can be determined by the obvious formula

$$\mathrm{Eff} = \frac{\gamma - \langle\gamma(z,\tau_0)\rangle}{\gamma - 1}, \tag{55}$$

where angle brackets denote averaging over all electrons entering the cross-section $z = 0$ during the period $2\pi/\omega$. Value (55) depends on coordinate, which allows determining the optimal amplifier length in terms of achieving maximum efficiency and output radiation power.

In the linear approximation, the dispersion equation (25) can be obtained from equations (54) and (50). Earlier, when presenting the linear theory, we proceeded not from solutions of the approximate equation (25), but from solutions of the exact dispersion equation (19), which is not accidental. The fact is that in the case of a high-density electron beam, the solutions of equations (19) and (25) differ, sometimes significantly, although they qualitatively coincide. Therefore, we always precede the solution of the nonlinear problem with linear analysis. Nonlinear solutions can be considered quantitatively reliable only in cases where the solutions of dispersion equations (19) and (25) are close. We will dedicate a separate work to the systematic analysis of nonlinear amplification modes. Here, we present some particular results characterizing possible values of amplification efficiencies and powers of amplified signals.

In Fig. 11 for a system with parameters $\omega_p = 20 \cdot 10^{10}$ rad/s, $r_0 = 1$ cm, $R = 3$ cm, $\varepsilon_d = 3$, $u = 2.27 \cdot 10^{10}$ cm/s, $\omega_b = 2.5 \cdot 10^{10}$ rad/s, the gain efficiencies at the frequencies of the eighth and fourteenth resonances are shown. The maximum efficiency of 15 to 25 percent is achieved at a length of 15–25 cm, after which it begins to decrease. As we can see, the gain efficiencies are quite high, and the optimal amplifier lengths are quite acceptable from an experimental point of view.

One of the important problems in developing Cherenkov amplifiers on high-current electron beams is the problem of radiation output through the output boundary of the beam-wave interaction region. The matching of the interaction region with the radiating device should be as good as possible. The electromagnetic field structure of high modes in the considered plasma-dielectric waveguides (see Fig. 3 and 9) suggests possible schemes of radiating devices. In the case of a waveguide in direct geometry, a conventional horn with a radius smaller than the waveguide radius can be used (Fig. 12). The walls of the waveguide and horn should be at the potential of the high-current accelerator anode. In this case, the surface connecting the waveguide to the horn simultaneously serves as a collector for the electron beam.

In the case of a waveguide in inverse geometry, a coaxial horn can be used (Fig. 13), such as those used in operating amplifiers on cable plasma waves [1–3]. The electron beam collector is the inner part of the coaxial. To equalize potentials, a jumper between the inner and outer parts of the coaxial horn is used, which makes the scheme less convenient compared to the direct geometry scheme.

8. CONCLUSION

In conclusion, let us formulate some findings from this work.

1. The use of waveguides with dielectric inserts allows the implementation of powerful single-mode (single-frequency) amplifiers with an operating frequency of about 10^{12} rad/s and even higher (wavelength: ~ 0.2 cm and even less). The efficiency of amplification at a length of ~ 20 cm can reach 15%. For an electron beam with a current of 1 kA and electron energy of 270 keV, this means an output power of about 40 MW.

2. The presence of plasma in waveguides with dielectric inserts should be considered undesirable. On one hand, in the sub-terahertz frequency range, the gain coefficient practically does not depend on the presence or absence of plasma. On the other hand, in the low-frequency region, due to the excitation of potential bulk Langmuir waves with high gain coefficient, low-frequency self-excitation of the emitter is possible, which will lead to suppression of gain in the sub-terahertz region.

3. The characteristic dependences of gain coefficients on frequency indicate the possibility of creating broadband amplifiers, including noise amplifiers, based on waveguides with dielectric inserts. Indeed, as can be seen from Fig. 2 and 8, the gain coefficient consists of a set of relatively narrow lines of approximately equal magnitude over a wide frequency range. There are reasons to believe that with an increase in the electron beam current, these individual lines broaden and merge into a single wide region of electromagnetic wave amplification.

4. The case of direct geometry of a waveguide with a dielectric insert is preferable compared to the case of inverse geometry due to the greater width of amplification zones.

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