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EVOLUTION OF ELECTROMAGNETIC FIELD PHASE OPERATORS PROPERTIES IN RABI AND JAYNES-CUMMINGS MODELS

A. V. Kozlovskii

Lebedev Physical Institute of the Russian Academy of Sciences, 119234, Moscow, Russia
e-mail: kozlovskiyav@lebedev.ru

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Abstract. The time evolution of mean values and dispersions of trigonometric functions of the quantum electromagnetic field phase operator interacting with a two-level atom has been studied. The field with a small number of photons is considered for various initial quantum states of the field and atom within the framework of Pegg-Barnett's Hermitian phase operator theory. The difference in phase operator evolution following from the Jaynes-Cummings theory and the Rabi model under conditions of ultrastrong atom-field coupling has been investigated. A qualitative difference between the results of the approximate Jaynes-Cummings model and the Rabi model is shown in the case of ultrastrong atom-field coupling for microscopic fields with photon numbers $\langle n \rangle \sim 1$ for Fock and coherent initial quantum states of the field and any initial states of the atom. It is shown that in the case of coherent initial field state with large $\langle n \rangle > 10$ under ultrastrong coupling conditions, the evolution of means and dispersions of field phase operators is characterized by a pronounced quantum effect of collapse and revival of the means and dispersions of these quantities.

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1. INTRODUCTION

The main fundamental physical process studied in quantum optics is the dynamics of interaction between an atom/molecule and a quantum mechanical electromagnetic field. In the study of such processes, important physical quantities that are studied (calculated or measured) are the quantum mechanical average populations of atomic quantum states (energy levels) and their fluctuations (dispersions), as well as the average values and fluctuations of population differences between the considered states of an atom or molecule. Other fundamental quantities characterizing the system of atoms and electromagnetic fields are the average values of field amplitude and their quantum fluctuations. The value of the complexvalued field amplitude (quantum mechanical average values of non-Hermitian creation/annihilation operators of the electromagnetic field) is described in quantum light theory using Hermitian photon number operators, as well as using Hermitian phase operator trigonometric function (POTF) of the electromagnetic field, which have real-valued average values and are thus directly measurable observable quantities.

Evolution (change in time) of the quantum mechanical state vector of the atom + field system $|\Psi(t)\rangle$ can be found for any initial state of the system $|\Psi(t=0)\rangle$ by solving the Schrödinger equation in the Rabi model. The Rabi model in the dipole approximation for a two-level atom accounts for both real atomic transitions with emission or absorption of field photons and virtual processes, meaning photon emission accompanied by atom excitation, as well as photon absorption accompanied by atomic transition to a lower energy state [1-4]. A widely used approximate theory based on the Rabi model is the Jaynes-Cummings model, within which the atom-field interaction Hamiltonian neglects terms responsible for virtual processes. The Jaynes-Cummings model (rotating wave approximation (RWA)) forms the basis of quantum laser theory. As calculations have shown [5-17], the applicability of RWA is limited to cases where the atom-field interaction constant is small compared to the field frequency. The predictions of RWA and NRWA for averages and dispersions of photon numbers and atomic level populations coincide only when the absolute value of the interaction constant $|g| < 10^{-2}\omega_f$, where ω_f — is the field frequency.

Currently, several experimental works [1-4] have demonstrated the possibility of creating an "artificial two-level atom" with an atom-field interaction constant $\tilde{g} \equiv |g|/\omega_f \sim 1$, i.e., values characteristic of ultra-strong coupling (USC) between atom and field. Under such conditions, as shown in theoretical works [5-17], RWA ceases to be valid for the dynamics of average photon number and atomic state populations.

In this work, we have investigated the evolution of average quantum mechanical quantities for POTF under USC conditions and compared the evolution of average values and quantum fluctuations of these operators for various initial quantum states of the electromagnetic field and two-level atom for NRWA and JCM.

The case of microscopic fields with small photon numbers is considered, i.e., fields currently used in experiments related to quantum information and quantum computing.

2. ELECTROMAGNETIC FIELD PHASE OPERATORS IN AN ARBITRARY QUANTUM STATE

In works [18,19] Pegg and Barnett considered the solution of equations for eigenfunctions of the phase variable in the discrete spectrum of phase eigenvalues. Calculations showed that the eigenvectors $|\theta_m\rangle$ of field phase operators, considered in a finite-dimensional basis of Fock states, for phase eigenvalues

$$\theta_m = \theta_0 + \frac{2\pi m}{S+1}, \quad m = 0,1,\ldots,S, \tag{1}$$

where $S+1$ — is an unlimitedly large but finite dimension of the Fock states basis, θ_0 — an arbitrary number determining the interval of change of phase eigenvalues ($\theta_0 \leqslant \theta_m \leqslant \theta_0 + 2\pi$), form a complete orthonormal basis of state vectors. In works [18-20], it was proposed to consider a discrete basis of phase state eigenvectors in $(S+1)$-dimensional subspace of Fock states for eigenvalues (5) in the form

$$|\theta_m\rangle = \frac{1}{\sqrt{S+1}}\sum_{n=0}^{S} e^{in\theta_m}|n\rangle. \tag{2}$$

The Hermitian phase operator $\hat{\varphi}_\theta$ with eigenvalues θ_m is thereby defined according to

$$\hat{\varphi}_\theta = \sum_{m=0}^{S}\theta_m|\theta_m\rangle\langle\theta_m|, \qquad \hat{\varphi}_\theta|\theta_m\rangle = \theta_m|\theta_m\rangle. \tag{3}$$

An important feature of the phase operator $\hat{\varphi}_\theta$, defined according to (1)-(3), is that the results of calculations of mean values and field phase dispersions qualitatively depend on the choice of parameter θ_0. Except for Fock states $|n\rangle$ and eigenfields of the phase operator $|\theta_m\rangle$, for which means and dispersions do not depend on θ_0, only the correct choice of parameter value θ_0 ensures obtaining physically meaningful results when calculating these mean values. Thus, in this theory, the form of the field phase operator depends on the considered quantum state of the field. At the same time, as shown in [18-21], fields (3) have the form

$$\cos\hat{\varphi}_\theta = \frac{e^{i\hat{\varphi}_\theta} + e^{-i\hat{\varphi}_\theta}}{2}, \quad \sin\hat{\varphi}_\theta = \frac{e^{i\hat{\varphi}_\theta} - e^{-i\hat{\varphi}_\theta}}{2i}$$

and can be written using relations

$$\begin{aligned} e^{i\hat{\varphi}_\theta} &= \sum_{n=1}^{S}|n-1\rangle\langle n| + e^{i(S+1)\theta_0}|S\rangle\langle 0|, \\ e^{-i\hat{\varphi}_\theta} &= \sum_{n=1}^{S}|n\rangle\langle n-1| + e^{-i(S+1)\theta_0}|0\rangle\langle S|, \end{aligned} \tag{4}$$

where the mean values of POTF $\cos\hat{\varphi}_\theta$ and $\sin\hat{\varphi}_\theta$ do not depend on parameter θ_0 and, consequently, do not depend on the specific quantum state of the field being considered. The dispersions of Hermitian field POTF also do not depend on θ_0.

3. RABI AND JAYNES-CUMMINGS MODELS OF DIPOLE INTERACTION OF A TWO-LEVEL ATOM WITH ELECTROMAGNETIC FIELD

Let's consider creation (annihilation) operators $\hat{a}^\dagger(\hat{a})$ of electromagnetic field, satisfying the following commutation relations: $[\hat{a},\hat{a}^\dagger] = 1$, and the photon number operator, which is defined using such operators according to $\hat{n} = \hat{a}^\dagger \hat{a}$.

In the case of dipole interaction between an atom and a field, the Hamiltonian of such a system in quantum theory can be written as (Hamiltonian NRWA for a two-level atom interacting with a single-mode electromagnetic field)

$$\hat{H} = \hbar\omega_f \hat{a}^\dagger \hat{a} + \frac{\hat{\sigma}^z}{2}\hbar\omega_a + \hbar\hat{V}, \quad (5)$$

where the operator of dipole interaction between the atom and field is

$$\hat{V} = (g\hat{a}^\dagger + g^*\hat{a})(\hat{\sigma}_+ + \hat{\sigma}_-) = \\ = g\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_- g^* + g\hat{a}^\dagger\hat{\sigma}_+ + \hat{a}\hat{\sigma}_- g^*, \quad (6)$$

for atomic subsystem operators

$$\hat{\sigma}^z = |e\rangle\langle e| - |g\rangle\langle g|, \\ \hat{\sigma}_+ = |e\rangle\langle g|, \quad \hat{\sigma}_- = |g\rangle\langle e|$$

and the constant of dipole interaction between field and atom

$$g = \sqrt{\frac{\omega_f}{2\hbar\varepsilon_0 V}}d. \quad (7)$$

Here $d \equiv \langle g|\hat{d}|e\rangle = |d_{eg}|e^{i\varphi_d}$ — is the matrix element of the atomic dipole transition, which is generally a complex number; V is the quantization volume of the electromagnetic field, ω_f — is the field frequency.

In the interaction representation, the Hamiltonian of atom-field interaction takes the form

$$\hat{V}_I = |g|(\hat{a}\hat{\sigma}_+ e^{-i\Delta_- t} + \hat{a}^\dagger\hat{\sigma}_- e^{i\Delta_- t} + \\ + \hat{a}^\dagger\hat{\sigma}_+ e^{i\Delta_+ t} + \hat{a}\hat{\sigma}_- e^{-i\Delta_+ t}), \quad (8)$$

and the Schrödinger equation for the state vector of the atom + field system can be written in the interaction representation as follows:

$$i\frac{d}{dt}|\Psi(t)\rangle = \hat{V}_I|\Psi(t)\rangle. \quad (9)$$

We will solve the equation of motion (9) using the following expansion of the system state vector in terms of the complete basis of Fock field states $|n\rangle$ and the basis of quantum states of the atom; excited $|e\rangle$ and lower $|g\rangle$ energy states of the two-level atom:

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty}[C_{e,n}(t)|e,n\rangle + C_{g,n}(t)|g,n\rangle], \\ C_{g,0}(t) = 0. \quad (10)$$

The equations for probability amplitudes included in (10) have the form

$$\frac{dC_{e,n}(t)}{dt} = -i|g|[\sqrt{n+1}\,e^{i\Delta_- t - i\varphi_d}C_{g,n+1}(t) + \\ + \sqrt{n}\,e^{i\Delta_+ t + i\varphi_d}C_{g,n-1}(t)], \\ \frac{dC_{g,n}(t)}{dt} = -i|g|[\sqrt{n+1}e^{-i\Delta_- t + i\varphi_d}C_{e,n+1}(t) + \\ + \sqrt{n}\,e^{-i\Delta_+ t - i\varphi_d}C_{e,n-1}(t)], \quad (11)$$

where denoted $\Delta_- \equiv \omega_a - \omega_f$, $\Delta_+ \equiv \omega_f + \omega_a$, $g = |g|e^{i\varphi_d}$.

Let's introduce dimensionless parameters

$$\tilde{\Delta}_- \equiv \frac{\omega_a - \omega_f}{|g|}, \quad \tilde{\Delta}_+ \equiv \frac{\omega_f + \omega_a}{|g|},$$

and also

$$\Omega_n \equiv \sqrt{\tilde{\Delta}_-^2 + 4(n+1)}, \quad \Omega_{n-1} \equiv \sqrt{\tilde{\Delta}_-^2 + 4n}, \\ t_g \equiv |g|t.$$

Let's further assume that the field frequency generally does not coincide with the atom transition frequency and the constant g of field-atom interaction is a complex number, and in the Hamiltonian of atom-field interaction (8), the last two terms responsible for virtual transitions are equal to zero (rotating wave approximation). Then the Schrödinger equation for an atom in the field can be solved analytically. The exact analytical solution of the equation system in the rotating wave approximation RWA (expansion coefficients in Fock states $C_{e,n}(t)$ and $C_{g,n}(t)$) can be written as [[22]

$$C_{e,n}(t_g) = C_{e,n}(0)A_n(t_g) - C_{g,n+1}(0)B_n(t_g), \\ C_{g,n}(t_g) = C_{g,n}(0)A_{n-1}^*(t_g) + C_{e,n-1}(0)B_{n-1}^*(t_g), \quad (12)$$

where it is denoted

$$A_n(t_g) \equiv \left[\cos\frac{\Omega_n t_g}{2} - \frac{i\tilde{\Delta}_-}{\Omega_n}\sin\frac{\Omega_n t_g}{2}\right]e^{i\tilde{\Delta}_- t_g/2}, \\ B_n(t_g) \equiv 2i\frac{\sqrt{n+1}}{\Omega_n}\sin\frac{\Omega_n t_g}{2}e^{i\varphi_d}e^{i\tilde{\Delta}_- t_g/2}. \quad (13)$$

4. DYNAMICS OF QUANTUM MECHANICAL AVERAGES AND FLUCTUATIONS OF TRIGONOMETRIC FIELD PHASE OPERATORS

According to the theory of Hermitian field phase operator $\hat{\varphi}$ [18, 19, 20, 21] (the lower index θ in the phase operator notation will be omitted hereafter) the quantum mechanical average values of POTF of the field for any quantum field state $|\Psi(t)\rangle$ have the form

$$\begin{aligned} \langle\cos\hat{\varphi}(t)\rangle &= \langle\Psi(t)\,|\cos\hat{\varphi}|\,\Psi(t)\rangle = \\ &= \mathrm{Re}\sum_{n=0}^{\infty}[C_{e,n}^{\ \ *}(t)C_{e,n+1}(t) + \\ &\quad + C_{g,n}^{\ \ *}(t)C_{g,n+1}(t)], \\ \langle\sin\hat{\varphi}(t)\rangle &= \langle\Psi(t)\,|\sin\hat{\varphi}|\,\Psi(t)\rangle = \\ &= \mathrm{Im}\sum_{n=0}^{\infty}[C_{e,n}^{\ \ *}(t)C_{e,n+1}(t) + \\ &\quad + C_{g,n}^{\ \ *}(t)C_{g,n+1}(t)]. \end{aligned} \tag{14}$$

For the mean squares of POTF, necessary for calculating dispersions (fluctuations) of these quantities, for arbitrary field and atom states $|\Psi(t)\rangle$ we find

$$\begin{aligned} \langle(\cos\hat{\varphi})^2(t)\rangle &= \langle\Psi(t)\,|\cos^2\hat{\varphi}|\,\Psi(t)\rangle = \\ &= \frac{1}{2}+\frac{1}{2}\mathrm{Re}\sum_{n=0}^{\infty}[C_{e,n}^{\ \ *}(t)C_{e,n+2}(t) + \\ &\quad + C_{g,n}^{\ \ *}(t)C_{g,n+2}(t)], \\ \langle(\sin\hat{\varphi})^2(t)\rangle &= \langle\Psi(t)\,|\sin^2\hat{\varphi}|\,\Psi(t)\rangle = \\ &= \frac{1}{2}-\frac{1}{2}\mathrm{Re}\sum_{n=0}^{\infty}[C_{e,n}^{\ \ *}(t)C_{e,n+2}(t) + \\ &\quad + C_{g,n}^{\ \ *}(t)C_{g,n+2}(t)]. \end{aligned} \tag{15}$$

Using relations (15) allows us to find expressions for dispersions (fluctuations) of POTF $\langle(\Delta\cos\hat{\varphi})^2(t)\rangle \equiv \langle(\cos\hat{\varphi})^2(t)\rangle - \langle\cos\hat{\varphi}(t)\rangle^2$ and $\langle(\Delta\sin\hat{\varphi})^2(t)\rangle \equiv \langle(\sin\hat{\varphi})^2(t)\rangle - \langle\sin\hat{\varphi}(t)\rangle^2$ through numerical solution of the system of coupled differential equations (11) within the framework of FM.

5. FOCK INITIAL STATE OF THE FIELD

Let's consider the case when the initial field at $t = 0$ is in a pure Fock state $|n_0\rangle$. In this case, generally, the initial values of the expansion coefficients of the system state vector are non-zero $C_{s,n_0}(0) \neq 0$ for $s = e, g$, while all others $C_{s,n_0}(0) = 0, n \neq n_0$.

In this case, the JCM solution (12), (13), using the RWA approximation, takes the following form:

$$\begin{aligned} C_{e,n_0}(t_g) &= C_{e,n}(0)\,A_{n_0}(t_g), \\ C_{e,n_0-1}(t_g) &= -C_{g,n}(0)\,B_{n_0-1}(t_g), \\ C_{g,n_0}(t_g) &= C_{g,n_0}(0)\,A^*_{n_0-1}(t_g), \\ C_{g,n_0+1}(t_g) &= C_{e,n_0}(0)\,B^*_{n_0}(t_g). \end{aligned} \tag{16}$$

The remaining $C_{e,n}(t_g) = C_{g,n}(t_g) = 0$ for $n \neq n_0, n_0-1$ or $n \neq n_0, n_0+1$ espectively.

For dimensionless time in (16), the following notation is used $t_g \equiv |g|\,t$.

If the initial state of the atom is $|e\rangle$, then the non-zero expansion coefficients of the system state vector are

$$\begin{aligned} C_{e,n_0}(t_g) &= C_{e,n}(0)\,A_{n_0}(t_g), \\ C_{g,n_0+1}(t_g) &= C_{e,n_0}(0)\,B^*_{n_0}(t_g). \end{aligned} \tag{17}$$

If the initial state of the atom is $|g\rangle$, then the time dependencies that are non-zero are

$$\begin{aligned} C_{e,n_0-1}(t_g) &= -C_{g,n}(0)\,B_{n_0-1}(t_g), \\ C_{g,n_0}(t_g) &= C_{g,n_0}(0)\,A^*_{n_0-1}(t_g). \end{aligned} \tag{18}$$

Using formulas (16)–(18) and (14), it is easy to verify that in the case of the initial Fock state of the field for the initial atomic states $|e\rangle$ and $|g\rangle$ the quantum mechanical averages of POTF (14) are equal to zero: $\langle\cos(\hat{\varphi}(t))\rangle_{n,RWA} = \langle\sin(\hat{\varphi}(t))\rangle_{n,RWA} = 0$, which corresponds to a uniform distribution of random field phase values from 0 to 2π for any moment in time. Similarly, from (16)-(18) and (15), we obtain that the dispersions $\langle(\Delta\cos\hat{\varphi})^2(t)\rangle_{n,RWA} = \langle(\Delta\sin\hat{\varphi})^2(t)\rangle_{n,RWA} = 1/2$ do not change over time for Fock initial field states and for initial atomic states $|e\rangle$ or $|g\rangle$.

Fig. 1*a*,*b* shows the time dependencies of dispersions $\langle(\Delta\cos\hat{\varphi})^2(t)\rangle_{n,RWA}$ and $\langle(\Delta\cos(\hat{\varphi}(t)))^2\rangle_{n,MR}$ (i.e., for RWA and NRWA respectively) under USS conditions for the initial state $|g\rangle$ and $|e\rangle$ of the

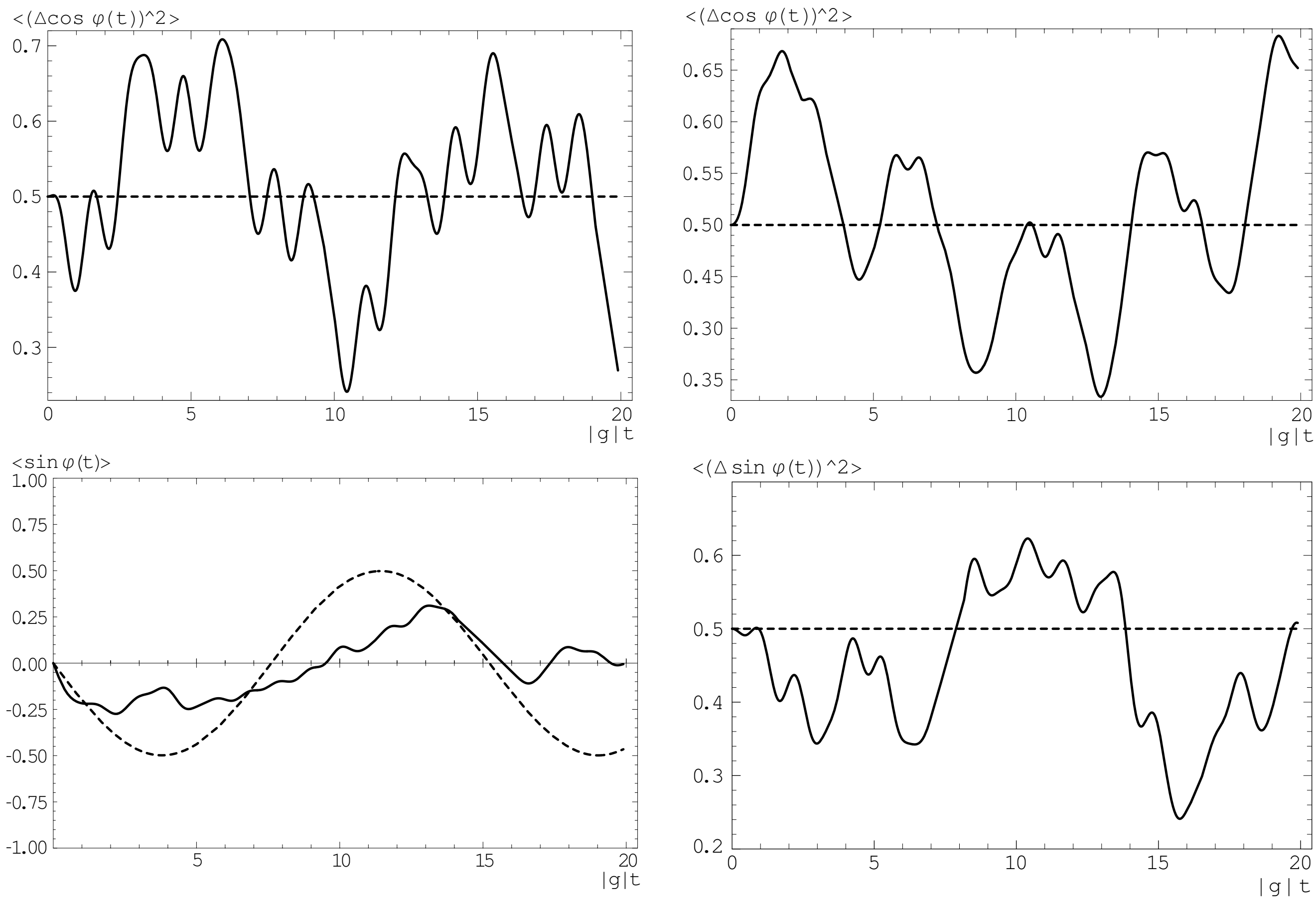

Fig. 1. *a* — Time dependence of the field phase cosine operator dispersion $\langle(\Delta\cos\hat{\varphi})^2\rangle$, following from the Rabi model for the atom + field system in the initial Fock state of the field ($|1\rangle$) and unexcited state of the atom $|g\rangle$, for the value of dimensionless coupling constant $g/\omega_f = 0.5$. The phase angle value of the transition matrix element $\varphi_d = 0$. The dashed line shows a similar dependence obtained within the Jaynes-Cummings model. *b* – Time dependence of the field phase cosine operator dispersion $\langle(\Delta\cos\hat{\varphi})^2\rangle$ for the atom + field system in the initial Fock state of the field ($|1\rangle$) and excited state of the atom $|e\rangle$ for the same parameter values. *c* – Time dependence of the mean value of the Pegg-Barnett field phase sine operator $\langle\sin\hat{\varphi}\rangle$, following from the Rabi model for the atom + field system in the initial Fock state of the field ($|n=1\rangle$) and superposition state of the atom $\frac{1}{\sqrt{2}}(|e\rangle+|g\rangle)$ for the value of dimensionless coupling constant $g/\omega_f = 0.5$. The phase angle value of the transition matrix element $\varphi_d = 0$. The dashed line shows a similar dependence obtained within the Jaynes-Cummings model. *d* – Time dependence of the field phase sine operator dispersion $\langle(\Delta\sin\hat{\varphi})^2\rangle$ for the atom + field system in the initial field state ($|n=1\rangle$) and atom superposition state for the phase angle value of the transition matrix element $\varphi_d = \pi/2$ with the same values of other parameters. The dashed line shows a similar dependence obtained within the Jaynes-Cummings model

atom and initial field state $|1\rangle$. The figure shows that the results of the two models contradict each other. RWA predicts that the POTF field dispersions do not change over time during atom-field interaction, while NRWA indicates complex time dependence of these quantities under USC conditions. The figures also show (cf. Fig. 1*a* and Fig. 1*b*) that the nature of dispersion evolution over time qualitatively depends on the initial state of the atom.

Calculations also show that the mean values of POTF coincide under such conditions for both models and are equal to zero for any moment in time.

The calculation results do not depend on the phase value of the transition matrix element φ_d.

A qualitatively different time dependence of means and dispersions of field phase operators is characteristic for the case of initial superposition state of excited and lower energy states of the atom:

$$|\psi_a(t=0)\rangle = C_e\,|e\rangle + C_g\,|g\rangle. \quad (19)$$

Fig. 1*c,d* shows examples of dependencies of mean values of the field phase sine operator for the initial atom superposition state and USC. As seen in Fig. 1*c*, the mean of the phase sine operator is non-zero for both RWA and NRWA and has a complex time dependence in NRWA in the USC regime. These dependencies differ qualitatively for the two models. RWA predicts regular changes in the means of phase operators over time, similar to Rabi oscillations.

As shown in Fig. 1*d*, the dispersions (fluctuations) of the sine operator vary with time within the NRWA framework and remain highly accurate and unchanged in the RWA theory for the considered case of initial condition of atomic states superposition and Fock state of the field. In this case, the mean value of the sine operator remains close to zero for any moment in time. The change in the phase angle value of the matrix element for transition between atomic states φ_d leads to a sharp increase within RWA of the oscillation amplitudes in time of the mean value of the phase sine operator and its dispersion (fluctuations) and their significant deviation from 0 and 1 / 2 respectively..

Note that in the interval of normalized dimensionless time variation $\Delta(|g|t) > 1000$ the considered means and dispersions take on a chaotic appearance within the intervals of these values' changes.

6. COHERENT INITIAL STATE OF THE FIELD

Let us consider a coherent state as the quantum state of the measured field

$$|\alpha\rangle = e^{-n_\alpha/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \quad \alpha = \sqrt{n_\alpha}e^{i\varphi_\alpha}, \quad n_\alpha \equiv |\alpha|^2 .$$

As seen in Fig. 2*a*, the time variation of the mean value $\langle\cos(\hat{\varphi}(t))\rangle_{\alpha,NRWA}$, calculated within the NR framework (NRWA) for the initial coherent field state $|\alpha\rangle$ and excited atomic state $|e\rangle$, exhibits complex irregular time dependence and does not show the character of standard regular Rabi oscillations. In Fig. 2*a* the dynamics of the mean value of the field phase operator is compared with the similar dependence $\langle\cos(\hat{\varphi}(t))\rangle_{\alpha,RWA}$, obtained using the RWA approximation. The figure shows that in our case of ultra-strong coupling, NRWA gives qualitatively different time dependence of the field phase operator means throughout the entire time interval except for negligibly small values $|g|t$. Fig. 2*b* shows the dependencies of dispersions $\langle(\Delta\cos(\hat{\varphi}(t)))^2\rangle_{n,NRWA}$

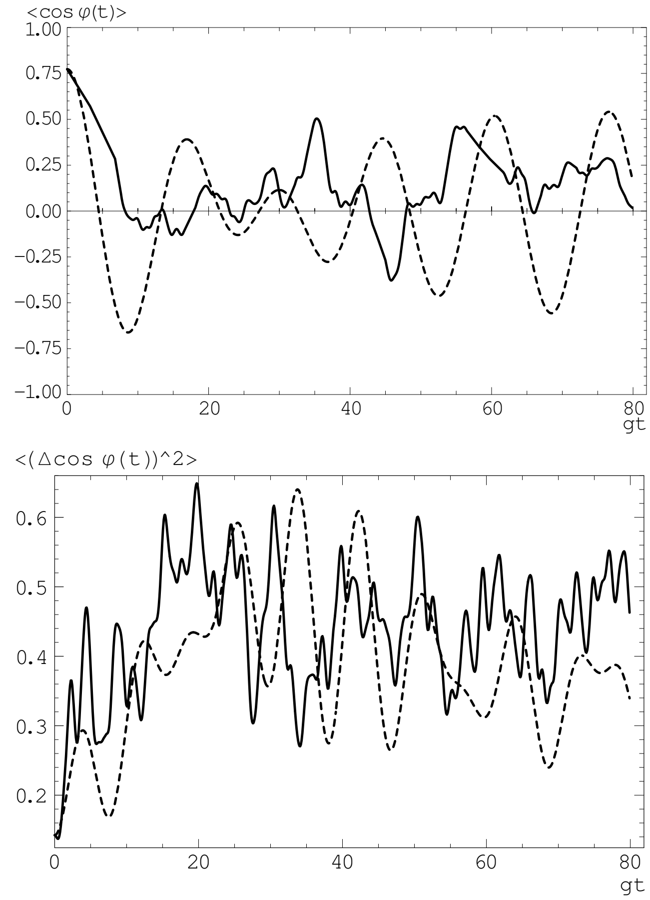

Fig. 2. *a* — Time dependencies of the mean value of the Pegg-Barnett field phase cosine operator $\langle\cos\hat{\varphi}\rangle$, following from the Rabi model for the atom + field system in the initial coherent field state ($\alpha = 1\rangle$) and excited atomic state $|e\rangle$, for the value of dimensionless coupling constant $g / \omega_f = 0.7$, $\omega_A = 1.1\omega_f$. The value of the phase angle of the transition matrix element $\varphi_d = 0$. The dashed line shows a similar dependence obtained within the Jaynes-Cummings model. *b* – Time dependencies of the variance of the field phase cosine operator $\langle(\Delta\cos\hat{\varphi})^2\rangle$ for the atom + field system in the initial coherent field state ($|\alpha = 1\rangle$) and excited atomic state $|e\rangle$ for the same parameter values

and $\langle(\Delta\cos(\hat{\varphi}(t)))^2\rangle_{n,RWA}$, obtained within NRWA and RWA frameworks respectively, for the same parameter values. Calculations show that, similar to the case of quantum mean values of field, under USC conditions the results of the two models are qualitatively different. The RWA approximation, which takes place in the Jaynes-Cummings theory, proves to be invalid for the case of USC of the atom with the field.

Our calculations have shown that the absolute values of the means and variances of the POTF field qualitatively depend on the initial value of the phase angle of the initial coherent field state φ_α (see Fig. 2 and Fig. 3 for comparison). Thus, the very nature of the evolution of mean values and variances of the

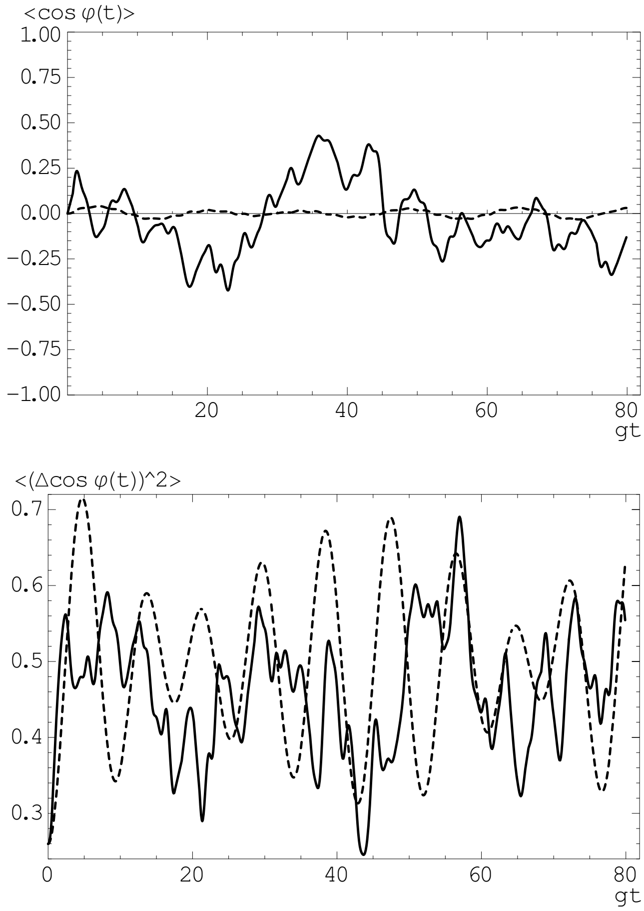

Fig. 3. *a* — Time dependencies of the mean value of the cosine phase operator in Pegg-Barnett field theory $\langle \cos\hat{\varphi} \rangle$, following from the Rabi model (NRWA) for the atom + field system in the initial coherent field state ($\alpha = e^{i\pi/2}\rangle$) and excited atomic state $|e\rangle$, for the value of dimensionless coupling constant $g / \omega_f = 0.7$, $\omega_A = 1.1\omega_f$. The phase angle value of the transition matrix element $\varphi_d = 0$. The dashed line shows a similar dependency obtained within the Jaynes-Cummings model (RWA). *b* – Time dependencies of the field phase cosine operator dispersion $\langle (\Delta\cos\hat{\varphi})^2 \rangle$ for the atom + field system in the initial coherent field state ($|\alpha = e^{i\pi/2}\rangle$) and excited atomic state $|e\rangle$ for the same parameter values

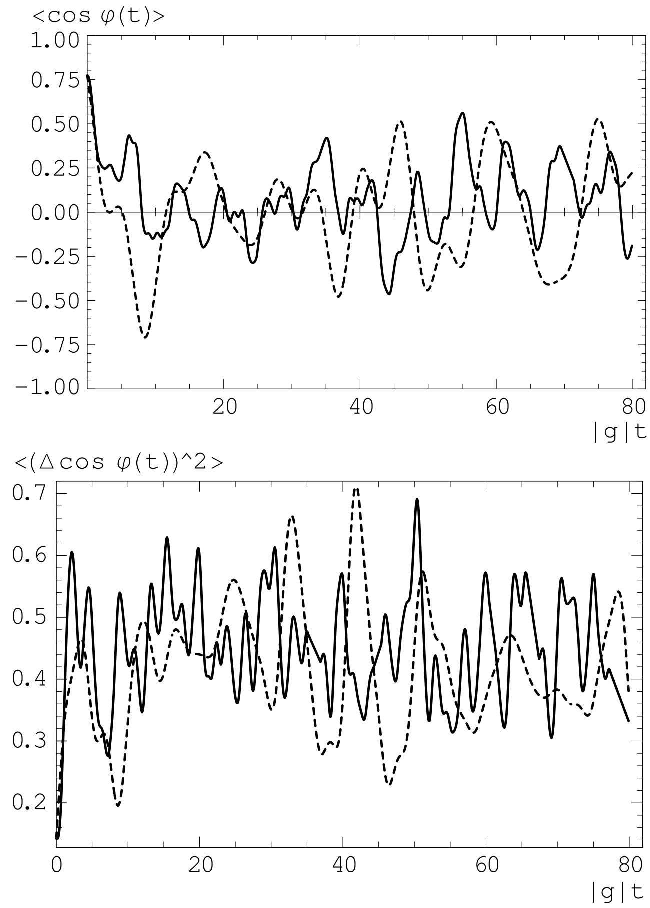

Fig. 4. *a* — Time dependencies of the mean value of the Pegg-Barnett field phase cosine operator $\langle \cos\hat{\varphi} \rangle$, following from the Rabi model (NRWA) for the atom + field system in the initial coherent field state ($|\alpha = 1\rangle$) and superposition atomic state $\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$, for the value of dimensionless coupling constant $g / \omega_f = 0.7$, $\omega_A = 1.1\omega_f$. The phase angle value of the transition matrix element $\varphi_d = \pi / 2$. The dashed line shows a similar dependence obtained within the Jaynes-Cummings model (RWA). *b* – Time dependencies of the field phase cosine operator dispersion $\langle (\Delta\cos\hat{\varphi})^2 \rangle$ for the atom + field system in the initial coherent field state ($|\alpha = 1\rangle$) and superposition atomic state $\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ for the same parameter

POTF field over time depends on the initial value of the phase angle of the coherent field state.

At the same time, as our calculations have shown, the dynamics of the means and variances of the field phase for initial atomic states $|g\rangle$ or $|e\rangle$ does not depend on the value of the phase angle of the transition matrix element and φ_d. The qualitative dependence on this parameter arises in the case of the initial atomic state superposition of the form (19). Fig. 4 shows the time dependencies of the mean and variance of the cosine phase operator for $\varphi_d = \pi / 2$ in the case of initial atomic superposition state $\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ and coherent field state. Calculations showed that for $\varphi_d = 0$ and the same system parameter values, the dynamics of the considered means and variances qualitatively differs from the dynamics in the case of $\varphi_d = \pi / 2$ under USC conditions.

With an increase in the number of photons of the initial coherent state n_α the difference between the results between RWA and NRWA theories for field evolution decreases for both mean values and dispersions of trigonometric phase operators and for mesoscopic coherent states with $n_\alpha \gg 1$ practically coincide for USS.

In the case of coherent initial field state at $n_\alpha \gg 1$ and atom state superposition (19), there is a pronounced phenomenon of collapse and revival for both mean values and dispersions of POTF field. Thus, at $n_\alpha = 30$ ($|\tilde{g}| = 0.1$, $\tilde{\Delta}_- = 1$) and in the interval of dimensionless time $700 < |g|t < 1100$ for mean values operator values satisfy the relations

$$\langle\cos(\hat{\varphi}(t))\rangle_{n,RWA/NRWA}, \langle\sin(\hat{\varphi}(t))\rangle_{n,RWA/NRWA} \approx 0,$$

$$\langle(\Delta\cos(\hat{\varphi}(t)))^2\rangle_{n,NRWA}, \langle(\Delta\cos(\hat{\varphi}(t)))^2\rangle_{n,RWA} \approx \frac{1}{2}$$

(same for operators sin). At $|g|t < 700$ damped regular Rabi oscillations occur. With increasing interaction time $|g|t > 1100$ oscillations of these quantities recover, i.e., the collapse phenomenon is replaced by the revival effect of Rabi oscillations [14,17,22-27]. At $|g|t < 700$ damped oscillations occur for both means and dispersions. It should be noted that the values of means and dispersions of the field POTF in the time variation interval, where the quantum collapse effect is present, are characteristic for the Fock state of the field.

The collapse and revival effect of oscillations of the considered quantities occurs for any initial atomic states.

The frequency of temporal oscillations of the considered quantities significantly differs from the dominant Rabi oscillation frequency $\Omega_R = \sqrt{\Delta^2 + 4g^2(n_\alpha + 1)}$, $\Delta \equiv \omega_a - \omega_f$, characteristic for Rabi oscillations of atomic population inversion for the initial coherent field state [14,17,27].

From the above, it can also be concluded that for studying the evolution of mean values and dispersions of field TPOF within the Pegg-Barnett phase operator theory, both the JCM (RWA) and RM (NRWA) approximations are applicable for meso- and macrofields with a large number of photons under USC conditions.

For a small number of photons in the coherent field state $n_\alpha \sim 1$ under USC conditions, the evolution character of mean values and dispersions of field POTF qualitatively depends on the phase angle of the coherent initial field state φ_α for any initial atomic state, while regular Rabi oscillations and the collapse and revival effect are absent.

The dependence of the dynamics of changes in these quantities on the phase angle of the atomic transition dipole moment φ_d occurs only in the case of the initial atomic state (19) for small values of $n_\alpha \sim 1$ (see Fig. 4), while for large $n_\alpha > 10$ the evolution of mean values and variances of field POTF is practically independent of φ_d for any initial atomic states. It should be noted that the evolution of means and variances of phase operators weakly depends on the initial state of the atom in the case of large photon numbers $n_\alpha > 10$ of the initial coherent field state.

7. CONCLUSION

In this work, we investigated the time evolution of quantum mechanical mean values (observables) and variances (quantum fluctuations) of field POTF during field-atom interaction. The case of USC of atom and field was considered and analyzed. The quantum mechanical Rabi model and Pegg-Barnett hermitian phase operator theory of electromagnetic field were used for calculations. The case of quantum microfields with mean photon number ~ 1 was considered. A comparison was made between the evolution patterns of these quantities following from the Rabi theory and the approximate Jaynes-Cummings theory using the rotating wave approximation under USC conditions.

The analysis was conducted for various initial quantum states of the field and two-level atom.

If the electromagnetic field is initially in a Fock state, the NRWA results fundamentally differ from the calculation results of the evolution of means and/or variances of field POTF under USC conditions. We have shown that in the case of the initial atomic state being in the excited or lower energy state, the phase operator variances within the RWA remain constant over time, while NRWA predicts a complex time dependence of field POTF variances. For the initial superposition state of two atomic states, the qualitative difference between NRWA and RWA results is present for both mean values and operator variances. In this case, there is a qualitative dependence of the evolution pattern of means and fluctuations of field POTF on the phase angle of the matrix element of the dipole transition between atomic states.

We have shown the inapplicability of RWA (rotating wave approximation, RWA) for calculating the evolution of phase operators under USC conditions for Fock initial field states.

If the initial field state is coherent $|\alpha\rangle$ with a small number of photons $n_\alpha \sim 1$, then the RWA (RWM) approximation proves to be inapplicable for calculating the dynamics of means and variances of POTF field under USC conditions for any initial state of the atom interacting with the field. With the increase of n_α the difference between NRWA and RWM results decreases and at $n_\alpha \gg 10$ the results of both models are almost indistinguishable for any initial atomic states under USC conditions.

This work shows that the evolution of the considered quantities qualitatively depends on the phase angle of the initial coherent state of the microscopic field φ_α for any initial atomic states, and under USC conditions qualitatively depends on the value of the phase angle φ_d of the atomic transition matrix element in the case of initial superposition of atomic states.

It is shown that in the case of initial coherent field state with the number of photons $n_\alpha \gg 1$ and for arbitrary initial atomic state, the time evolution of mean values and variances of field POTF is characterized by the phenomenon of collapse and revival of Rabi oscillations of these quantities.

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